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Sources and uses of knowledge in a dynamic network technology

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Abstract

We estimate a dynamic network technology where new knowledge in the form of publications in STEM (science, technology, engineering, and mathematics) is an intermediate product. Knowledge is produced in the first stage of production and is used in the second stage of production to produce a final output of real consumption, which equals gross domestic product minus investment spending on physical capital minus research and development expenditures. Knowledge also spills over between producers as it becomes disseminated. The two stages of production are linked between periods as investments in research capital and physical capital enhance future production possibilities. Our model combines several theories of production: dynamic data envelopment analysis (DEA) and two-stage network DEA. Using pooled data on 53 countries during 1999–2012, the model estimates indicate that dynamic efficiency averages about 70%. Countries could increase final consumption by about 25% via greater technical efficiency in production and by another 5% via an optimal intertemporal reallocation of investment spending.

Keywords: dynamic network; intermediate products; knowledge

1. Introduction

Since the work of Solow (1956), there has been much research on the optimal path of investment spending and its relation to productivity growth. Productivity growth contributes to economic

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well-being as more outputs are produced using fewer inputs. Such growth occurs when producers become more efficient at using a known technology or when technical change occurs. Here we model the sources and role of knowledge in productivity growth, focusing specifically on knowledge production by researchers in STEM (science, technology, engineering, and math). The National Science Foundation (NSF) was started in 1945 by President Roosevelt to promote research and education in STEM fields (Rothwell, 2013). Research and development (R&D) in STEM fields has been shown to be an important driver of productivity growth, with estimates that R&D growth contributes between 30% and 70% of productivity growth (Adams, 1990; Jones, 2002; Peri et al., 2015).

Research is undertaken by public sector organizations such as public universities and various government agencies and in the private sector by private universities, nonprofit organizations, and for-profit firms. When published, this new knowledge takes on characteristics of a public good, being both nonrival and nonexcludable. Adams (1990) found knowledge in science, engineering, and mathematics, and knowledge spillovers contributed over half the productivity growth in U.S. manufacturing industries from 1953 to 1980. Jones (2002) attributed 50–70% of labor productivity growth during 1950–1993 to greater research intensity as measured by the number of scientists and engineers engaged in R&D in G-5 countries. Peri et al. (2015) found that employment growth among foreign STEM researchers residing in the United States explained between 30% and 50% of productivity growth in the 100 largest U.S. cities.

In this paper, we seek to better understand how advances in the STEM disciplines contribute to economic growth and determine the optimal allocation of resources devoted to these areas. To do this, we develop a dynamic network model that incorporates STEM knowledge production as an intermediate product in the macroeconomy, allowing for knowledge spillovers across disciplines. New knowledge is used to produce final output, and this knowledge spills over and can be used by other producers. The optimization problem we develop and estimate seeks the maximum expansion in final outputs that can be achieved via greater production efficiency due to an optimal intertemporal reallocation of resources. Our model is related to the work of Romer (1990) who assumed that technological change occurs because of "intentional actions taken by people who respond to market incentives" (Romer, 1990, p. S72). Furthermore, we assume that the publications that are produced at a given point in time are initially excludable, but then become nonrival and nonexcludable in later periods. An important result of Romer (1990) is that capital will be underallocated to a research sector for two reasons: first, because private producers are unable to capture all of the benefits of research, and second, because of monopoly pricing of the private good. In addition to determining the degree to which an intertemporal reallocation of resources can expand final output, we also empirically investigate Romer's theory that resources are underallocated to R&D.

As researchers produce knowledge, not only does their own ability to produce knowledge expand, that knowledge also spills over to researchers at other universities and private organizations as it becomes more widely known. These knowledge spillovers, across disciplines (see Daraio et al., 2018), countries, and time increase the knowledge base and link research to private sector productivity. We consider these linkages as a form of network technology where knowledge is an intermediate product used to produce other goods and services. This dynamic network framework allows us to consider reallocations of knowledge resources through time in order to better capitalize on knowledge spillovers and enhance economic growth.

From a methodological point of view, we develop an efficiency analysis approach based on the estimation of efficient frontiers. This literature is constantly growing as it is witnessed by the increasing number of reviews published in the literature (for recent surveys, see, e.g., Narbón-Perpiñá and De Witte, 2018; Catalano et al., 2019; Daraio et al., 2020). Our model combines several theories of production: dynamic data envelopment analysis (DEA) and two-stage network DEA. Dynamic network analyses based on DEA can be found in Tran and Villano (2018), Moreno and Lozano (2018), Hsiao et al. (2019), and Chen et al. (2020).

We combine bibliometric data (e.g., publications, citations, authors) with aggregate data on public and private research investment, as well as other macroeconomic variables (e.g., real gross domestic product [GDP], employment), to construct an international panel for 53 countries and 18 STEM disciplines for the years 1996–2012. We use this panel to consider the potential reallocation of investment between research and industry, through the lens of our dynamic network model of linked research and industry production technologies. Consistent with the theoretical result of Romer (1990), we find that the optimal ratio of research investment to industrial investment generally exceeds that ratio in practice. Reallocations of investment spending across periods along with changes in the mix of investments in physical capital versus research capital offer the potential to expand GDP by an average of 5%.

We provide several contributions to the literature by building a two-stage network model with knowledge as an intermediate product; by accounting for knowledge spillovers between countries; by extending the network model to a dynamic model that allows two types of trade-offs between final consumption and investment spending, and between investment spending on physical capital and investment spending on research capital where the objective is to maximize the sum of final consumption across all periods; and finally, by offering some empirical evidence of Romer's theory that resources are underallocated to R&D.

In the next section, we overview the related literature on knowledge spillovers in more detail. We then provide some background on network production models in this context, before introducing our own dynamic network model of knowledge production in Section 3. We detail the application and results in Section 4 and conclude with a final discussion in Section 5.

2. Related literature

We develop a dynamic network model of production that accounts for international spillovers of knowledge. Knowledge transfers from the R&D sector to the industrial sector and back again are captured within our model. Even as knowledge is incorporated into new techniques by its user, that knowledge also spills over and affects other producers. Measuring knowledge is fraught with difficulties and this section also reviews research on the difficulties and limitations of designing meaningful knowledge indicators.

To measure knowledge spillovers requires consistent and widely available indicators of knowledge produced. One point to emerge from the literature concerns the measurement and quantification of knowledge itself. Bibliometric data on publications and citations have been recognized as knowledge outputs, but international comparisons of productivity in R&D are limited by a paucity of comparable data (Aksnes et al., 2017). This presents several challenges for cross-country analysis. First, one must correctly count publications/citations when co-authors reside in more than one country. Second, although citation indexes such as the Web of Science and Scopus provide fairly wide coverage of some fields such as the natural sciences, these indexes are more limited in their coverage of papers in the humanities, social sciences, and engineering. To the extent that countries realize a comparative advantage in these thinly covered fields, their performance estimates will be biased downward. Third, inquiries into the presence or absence of statistical power and bias in economics (Ioannidis et al., 2017) and other scientific research (Ioannidis, 2005) has found that many published results are a reflection of the prevailing bias and are likely false. Fourth, even if the above challenges are met, new research findings are necessarily heterogeneous, so that publication counts might not be an accurate measure of knowledge production (Foray, 2004). Finally, research grants received might be a more accurate measure of research quantity and quality since grants imply a willingness to pay for the research conducted (Johnes and Johnes, 1993; Agasisti and Haelermans, 2016).

Recent research on the scientific impact of nations identified the United States and United Kingdom as the top two leaders in highly cited publications from 1997 to 2001 (King, 2004), Iran and Saudi Arabia as scientific leaders in the Gulf states (Moed, 2016), and China as the fifth leading country in scientific publications with 6.5% of the world's total publications in 2004 compared to the 8.3% publications in the United Kingdom and 30.5% publications in the United States (Zhou and Leydesdorff, 2006). Moed and Galevi (2014) found that Iran and China had high growth rates in publications relative to other countries with similar levels of publications and that regional collaboration between countries in Asia acts as a stepping stone to greater international scientific collaborations.

Gralka et al. (2018) control for teaching and estimate the research efficiency of 72 German public universities using grants, publications, and highly cited publications as alternative measures of research output. They find a high positive correlation between the measures of efficiency, which they take "as a sign that universities which are good in acquiring third party funds are the ones which are equally good in publishing high-quality research" (Gralka et al., 2018, p. 6).

Another important point to emerge concerns the flow of knowledge between academic research and the private sector, commonly termed as "university–industry knowledge transfer." Our dynamic network model described in the next section can be specified to capture these linkages. One form of knowledge transfer occurs when university researchers collaborate with industry researchers. Tartari et al. (2014) find evidence that this collaboration is more likely when the departmental peers of a researcher also collaborate with industry and support this finding by arguing that researchers compare themselves and compete with their fellow departmental researchers. Perkmann et al. (2013) examine various aspects of academic engagement between universities and industry including collaborative research, contract research, consulting, and informal engagement and write that "academic engagement often precedes commercialization in time and can hence be regarded as an input factor to the latter. Academic engagement may also accompany commercialization, for instance when spin-off companies work collaboratively with the university labs they originated from" (Perkmann et al., 2013, p. 424). In a network model, such engagement would typically comprise a two-way flow where universities produce and receive intermediate products from industry. Jaffe (1989) provides evidence that university research causes industrial R&D, but not *vice versa*. Our aggregated country data do not allow us to distinguish between university publications and industry publications. However, if such data were available our network model can be easily adapted to represent the two-way flow between academic research and industry production.

In 1980, university–industry transfers in the United States received a boost from passage of the Bayh–Dole Act, which allowed universities to patent and license inventions from research that had received Federal funding. One purpose of the Act was to provide incentives for universities to engage in more research, with an intended outcome of increased patenting. However, Mowery and Sampat (2005) report that university–industry transfers had already increased in the pre-Bayh–Dole period and that lobbying by U.S. research universities contributed to passage of the Act. Thus, passage of Bayh–Dole might have been "as much an effect as a cause of expanded patenting and licensing by US universities" (Mowery and Sampat, 2005, p. 237). In the early 2000s, Germany, Austria, Denmark, Finland, and Norway ended the "professor's privilege" where university researchers enjoyed full rights to their inventions and intellectual property and moved toward patenting and licensing policies similar to Bayh–Dole. Hvide and Jones (2018) find a 50% decline in entrepreneurship and patenting among professors in Norway after the end of the "professor's privilege."

Since research dollars and time can be used to produce a mix of basic research and applied research, there was some concern that Bayh–Dole would tip the scale too far away from nonexcludable basic research toward applied research, which is more easily excludable. Weber and Xia (2011) and Fukuyama et al. (2016) review research on the applied/basic research trade-off. Some research supports the idea that universities gain monopoly power from licensing inventions and that monopoly power favors resources going into industrial applications (Boldrin and Levin, 2009; Just and Huffman, 2009; Weber and Xia, 2011). Other research finds that applied research and basic research are complements rather than substitutes (Thursby and Thursby, 2002; Fabrizio and DeMinin, 2008; Azoulay et al., 2009).

In a broader context than Bayh–Dole, Kealey (1996) even argues that many inventors experienced applied and economic success before scientists knew why their invention worked. Although our model incorporates only a money flow as the intermediate product, our dynamic model can also incorporate nonmonetary flows. For instance, even though patent holders get exclusive use of various types of applied research, scientists who cooperate with industry can learn more about whether their theories are validated (Perkmann et al., 2013). See also the classic paper by Comanor and Scherer (1969, p. 398) and their conclusion that "patent statistics may be a better index of research input than output." This further highlights the connections between academic and industry research. The next section outlines our use of network production theory to model these connections.

3. Network models

In practice, the production of final goods and services often occurs in stages, where output from one stage becomes an input in subsequent stages. As a simple example, the stages of production for a wooden chair might include harvesting the timber, milling the lumber, and then crafting the furniture. This linkage of production, via intermediate products (i.e., the timber and milled wood), constitutes a network production technology. Here, we draw on network production methods to model the formation of new knowledge, and in turn, its contribution to the production of other goods and services in the economy.

Before turning to our dynamic network model, we first introduce a static network model where knowledge is produced in the first stage of production and then subsequently used as an input in the second goods' producing stage using what are called distance functions to represent technology and assess performance. Distance functions are a generalization of production functions and have been used to represent multiproduct knowledge technologies, accommodating specification of many outputs and inputs.

These functions, along with duality theory, have been used to estimate shadow prices for knowledge spillovers between universities (Weber, 2019), to estimate elasticities of transformation between knowledge outputs of patents, Ph.D. students, and publications (Weber and Yin, 2011), and to simulate a reallocation of NSF funds given to enhance knowledge outputs at various U.S. institutions of higher education (Fukuyama et al., 2016). Distance functions can be estimated parametrically (Weber and Xia, 2011; Weber, 2019) or using nonparametric DEA (Fukuyama et al., 2016). We extend these models and use a dynamic network distance function to model the role that knowledge plays in a dynamic network technology.

3.1. Overview of previous network production models

Distance functions have been widely used to estimate producer performance. Early models were static black box representations of a production technology. More recent work has extended these models to account for network production where intermediate products are first produced and then used to produce final outputs. Intermediate products are a key feature of network models (Färe and Grosskopf, 1996a, 1996b, 2000; Prieto and Zofío, 2007; Bogetoft et al., 2009). Such products tend to be outputs of one division or production stage and inputs to another division or production stage. Kao (2014, 2017) lays out the theoretical foundation and reviews and extends the many forms that network models take. Bostian et al. (2018) examine interdisciplinary knowledge productivity spillovers among 16 STEM fields in a statistical network setting. They find higher interactions between some fields such as physics and materials science and between chemistry and chemical engineering, but low or zero interactions among other fields, such as biochemistry and computer science and between medicine and mathematics.

When the network technology extends over several periods, we call it a dynamic network technology. Färe et al. (2018) provide an overview of various types of dynamic network models. Fallah-Fini et al. (2014) identify five factors that lead to an intertemporal dependence between inputs and outputs: production delays, inventories, quasi-fixed factors such as capital, adjustment costs, and learning models. A decision-making unit (DMU) exhibits dynamic efficiency when it "cannot shift production from one period to another and generate a larger present value of the firm's utility" (Fallah-Fini et al., 2014, p. 53). See also the review of dynamic models by Mariz et al. (2018).

In education, a network might consist of a school district with primary and secondary school types where administrators seek to allocate their budget across school types and between schools of the same type so as to maximize some preferred educational outcome. Grosskopf et al. (2017) use a static network DEA model to simulate the effects of weighted-student funding on primary and secondary Texas' school districts. Shen et al. (2016) use an input/output network model to prioritize and determine which subfields of physics act as support or are supported by a given field. Using citations as the intermediate product, they find that there can be strong links between subfields (such as quantum mechanics and mechanic control) even when there are few direct citations between them. They also find that statistical physics supports many other subfields, but relativity has a much weaker influence on other subfields.

Our interest lies in two linked processes: the formation of new knowledge from existing knowledge and the subsequent role that new knowledge plays in the production of final goods and services in the economy. This first process can be considered the research stage, where we measure knowledge in terms of research publications and citation counts. In the research process, knowledge builds on the work of others, so that existing publications lead to new publications, with the latter citing the former. The second process can be considered the product development stage, where producers harness new knowledge flowing from the initial research stage to produce final goods and services in the economy. We consider these linked processes in a dynamic sense, the key distinction being the potential to reallocate resources in the economy between the two stages of production. Important resources include capital, decomposed into knowledge capital (e.g., lab space and equipment) and physical production capital (e.g., factories and machinery), and labor employed at each stage (e.g., research authors, manufacturing workers).

The dynamic aspect of the model allows producers to choose intermediate production at different points in time in order to maximize the weighted sum of final outputs over time. Thus, when technical progress is expected, producers might find it optimal to forgo final consumption in t and instead, produce the two intermediate products in order to expand future final consumption. Furthermore, standard microeconomic theory requires investments to be made to equalize the marginal products of physical capital and knowledge capital to future final output across time. Comparing the actual versus the optimal timing and quantities of the two intermediate products allows us to infer whether dynamic efficiency is increasing or decreasing. In addition, comparing the actual mix of the two intermediate products with the optimal mix can help inform policy makers about the relative marginal products of physical capital and knowledge capital.

Here we mention only a small subset of research that uses DEA in a dynamic framework. Building on the work of Ramsey (1928), Färe and Grosskopf (1996a, 1996b, 1998) and Sengupta (1996) develop dynamic models where current decisions on capital investment impact future production possibilities (see also Nemoto and Goto 2003; Silva and Stefanou, 2003; Ouellette and Yan, 2008; Serra et al., 2011; Sacoto et al., 2015). In an infinite horizon model, Lansink et al. (2015) and Silva et al. (2015) choose investment spending to minimize the present value of all future costs of production. Färe et al. (2012) and Fukuyama and Weber (2015) examine dynamic efficiency and productivity when undesirable outputs, such as pollution or nonperforming loans, are part of the technology. Fukuyama et al. (2016) use DEA to simulate the dynamic reallocation of NSF funds for nanobiotechnology across universities so as to maximize knowledge outputs of publications, patents, and Ph.D. students. In general, these studies found that efficiency gains could be realized by an intertemporal reallocation of resources. In the context of knowledge production, this intertemporal reallocation might be stymied by producers who are unable to capture all of the benefits of their own actions, such as occurs when knowledge is a public good or has positive spillover effects.

Table 1
Model notation

Static		Dynamic	
y	Real GDP	fy	Final consumption
L	Employment	iy_1	Research investments
A	Authors	iy_2	Physical capital investments
$c1$	Knowledge capital	γ	Depreciation of research capital
$c2$	Real physical capital	δ	Depreciation of physical capital
z_1	Own publications		
z_2	Own publication quality		
$\tilde{z}$	Spillover publications of others		

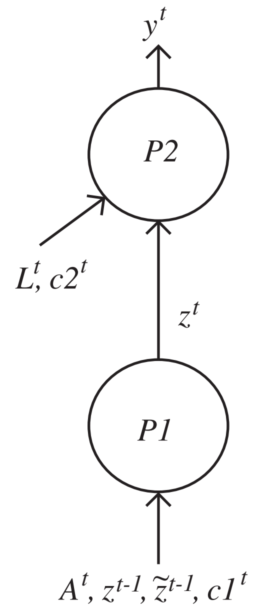

Fig. 1. Static technology.

3.2. The static network model

Before turning to the model, we introduce the associated notation (both static and dynamic) for reference in Table 1.

We begin with the static version of our aggregated network model before extending it to the dynamic case. Figure 1 illustrates a static network with two stages of production. In the first stage, country k produces new research knowledge, z_k^t, using authors, A_k^t, research capital, $c1_k^t$, and past research knowledge, where $t = 1, \ldots, T$ represents the time period that we take to be one year. Past research knowledge enters stage 1 in two ways: first, k draws on its own stock of knowledge from $t-1$ to produce new research in t. Second, the researchers in k also gain access to the research produced by researchers in other countries. This spillover effect input is $\tilde{z}_k^{t-1} = \sum_{j \neq k}^{K} z_j^{t-1}$. We distinguish between own past publications (z_k^{t-1}) and past external publications ($\tilde{z}_k^{t-1}$) because there are various geographic constraints that differentially affect the transmission of knowledge. For instance, a common language within a country provides easier communication of ideas within that country. Researchers within a country might also have access to a library consortium that shares materials. Dropping the k subscript, we write stage 1 production technology as

$$P1^t = \{z^t : \quad (A^t, c1^t, z^{t-1}, \tilde{z}^{t-1}) \text{ can produce } z^t\}. \tag{1}$$

In stage 2, real GDP, y^t, is produced using labor, L^t, real physical capital, $c2^t$, and the research knowledge produced in $P1$. Stage 2 output set is

$$P2^t = \{y^t : \quad (z^t, \ L^t, c2^t) \text{ can produce } y^t\}. \tag{2}$$

Combining (1) and (2) gives the network technology for t:

$$N^t = \{(A^t, c1^t, z^{t-1}, \tilde{z}^{t-1}, z^t, L^t, c2^t, y^t) : \quad z^t \in P1^t \text{ and } y^t \in P2^t\}. \tag{3}$$

The Shephard (1970) output distance function is used as a functional representation of the network technology set, N^t. The reciprocal of this function gives the maximum proportional expansion in outputs that would be feasible if the DMU were to become efficient and produce on the frontier of the network technology given by (3). This distance function is

$$D_o^t(A^t, c1^t, z^{t-1}, \tilde{z}^{t-1}, c2^t, L^t, y^t) = \min\left\{\phi : \ \frac{y^t}{\phi} \in N^t\right\} \tag{4}$$

and is bounded between 0 and 1. Efficient DMUs have $D_o^t(\cdot) = 1$ and inefficient DMUs have $D_o^t(\cdot) < 1$.

We use DEA to represent the network technology. DEA constructs a best-practice frontier from the observed inputs and outputs of the $k = 1, \ldots, K$ DMUs. The DEA method chooses intensity variables for the two stages, λ_{1k}^t and λ_{2k}^t, $k = 1, \ldots, K$, to make the largest convex set of feasible outputs and inputs such that the feasible inputs are no less than a linear combination of observed inputs and the feasible outputs are no greater than a linear combination of observed outputs. The DEA network technology set is

$$\begin{aligned}
&N^t = \{(A^t, c1^t, z^{t-1}, \tilde{z}^{t-1}, z^t, L^t, c2^t, y^t) : \\
&A^t \geq \sum_{k=1}^{K} \lambda_{1k}^t A_k^t, \quad c1^t \geq \sum_{k=1}^{K} \lambda_{1k}^t c1_k^t, \quad z^{t-1} \geq \sum_{k=1}^{K} \lambda_{1k}^t z_k^{t-1}, \\
&\tilde{z}^{t-1} \geq \sum_{k=1}^{K} \lambda_{1k}^t \tilde{z}_k^{t-1}, \quad z^t \leq \sum_{k=1}^{K} \lambda_{1k}^t z_k^t, \quad \lambda_{1k}^t \geq 0, \ k = 1, \ldots, K, \ t = 1, \ldots, T, \\
&z^t \geq \sum_{k=1}^{K} \lambda_{2k}^t z_k^t, L^t \geq \sum_{k=1}^{K} \lambda_{2k}^t L_k^t, \quad c2^t \geq \sum_{k=1}^{K} \lambda_{2k}^t c2_k^t, \\
&y^t \leq \sum_{k=1}^{K} \lambda_{2k}^t y_k^t, \quad \lambda_{2k}^t \geq 0, \ k = 1, \ldots, K, \ t = 1, \ldots, T.
\end{aligned} \tag{5}$$

The two stages of production are linked through the intermediate product of research knowledge. In stage 1, the research output constraint is $z^t \leq \sum_{k=1}^{K} \lambda_{1k}^t z_k^t$ and in stage 2 the research input constraint is $z^t \geq \sum_{k=1}^{K} \lambda_{2k}^t z_k^t$. Together these two constraints imply that

$$\sum_{k=1}^{K} \lambda_{1k}^t z_k^t \geq z^t \geq \sum_{k=1}^{K} \lambda_{2k}^t z_k^t, \quad t = 1, \ldots, T, \tag{6}$$

meaning that the intensity variables must be chosen so that stage 2 cannot use more of the input z^t than was produced in stage 1.

Using (5), the reciprocal of the output distance function is estimated as

$$D_o^t(A^t, c1^t, z^{t-1}, \tilde{z}^{t-1}, z^t, L^t, c2^t, y^t)^{-1} = \max_{\phi, \lambda_1, \lambda_2} \ \phi^{-1} \quad \text{subject to} \ \ y^t \phi^{-1} \in N^t, \tag{7}$$

where the choice variables in (7) are the expansion factor ϕ^{-1} and the intensity variables for the two stages of production, λ_{1k}^t and λ_{2k}^t, $k = 1, \ldots, K$.

3.3. Dynamic network model

We extend the static network model to a dynamic model by decomposing real GDP into final consumption and two types of investment spending. Real GDP equals the sum of final consumption, fy^t, and the two types of investment: $y^t = fy^t + iy1^t + iy2^t$, where $iy1^t$ are research investments that flow to stage 1 in $t + 1$ and $iy2^t$ are physical capital investments that flow to stage 2 in $t + 1$.[1] Investments made in t expand production possibilities in period $t + 1$. Thus, the producer has several decisions to make. First, should resources be used to produce current final output or future outputs? Second, the two types of investment alter subsequent production in different ways. Research investments, $iy1^t$, are an input to stage 1 in $t + 1$ while physical capital investments, $iy2^t$, are an input to stage 2 in $t + 1$. Thus, what is the relative optimal mix of the two intermediate products?

We assume that research capital ($c1$) depreciates at rate $(1 - \gamma)$. Therefore, the available research capital available in $t + 1$ equals the depreciated value of research capital from t plus $iy1^t$:

$$c1^{t+1} = \gamma c1^t + iy1^t. \tag{8}$$

Similarly, real physical capital ($c2$) depreciates at rate $(1 - \delta)$ so that available real physical capital in $t + 1$ equals the depreciated the amount of real physical from t plus $iy2^t$:

$$c2^{t+1} = \delta c2^t + iy2^t. \tag{9}$$

The dynamic network technology is

$$DN = \{(z^t, y^t) : \ \ (z^t, y^t) \in N^t, \ t = 1, \ldots, T\} \tag{10}$$

and is illustrated in Fig. 2.

[1]Our method can be extended to cases in which there are multiple final outputs.

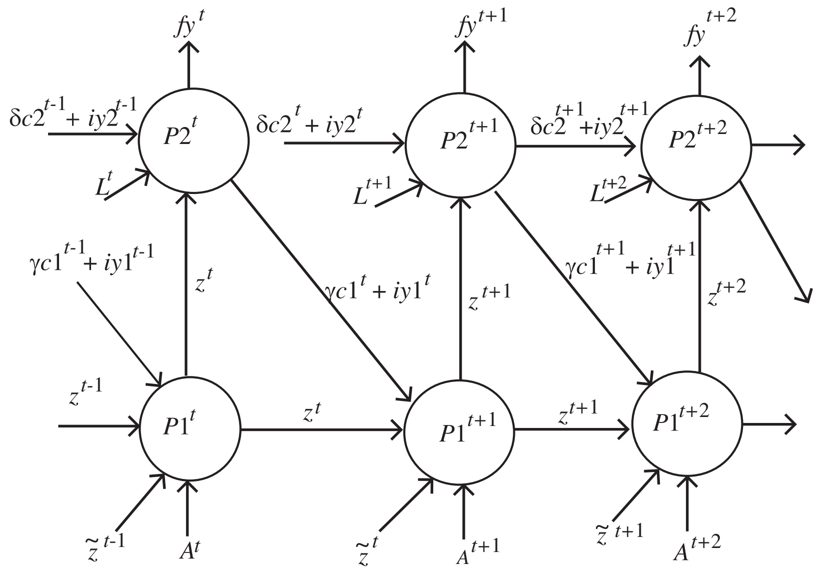

Fig. 2. Dynamic network technology.

Our dynamic optimization problem maximizes the sum of weighted final outputs across all periods by choosing the amounts of the two intermediate products:

$$\text{maximize } \sum_{t=1}^{T} \Psi^t \phi^{-1,t} \text{ subject to}$$
$$(z^t, \phi^{-1,t} y^t),\ t = 1, \ldots, T \ \in DN, \tag{11}$$

where Ψ^t, $t = 1 \ldots, T$ are predetermined weights that account for the DMU's rate of time preference. For instance, if the opportunity cost of capital is r, the weights might be chosen as $\Psi^t = 1/(1+r)^{t-1}$.

We impose initial and terminal conditions for research capital and real physical capital. For DMU o in $t = 1$, we take research capital and physical capital as given by their observed values, $c1_o^1$ and $c2_o^1$. In the terminal period, T, we take the investments in research capital and physical capital for DMU o as given by their observed values: $iy1_o^T$ and $iy2_o^T$. The optimization problem chooses two types of investment in $t = 1, \ldots, T-1$. Let these optimal intermediate products be $iy1^{t*}$ and $iy2^{t*}$, $t = 1, \ldots, T-1$. Thus, in $t = 2, \ldots, T$, the amounts of research capital and physical capital equal the depreciated value of the capital stock from the previous period plus investment. For instance, in $t = 2$ research capital equals $c1_o^2 = \gamma c1_o^1 + iy1^{1*}$ and physical capital equals $c2_o^2 = \delta c2_o^1 + iy2^{1*}$. In $t = 3$, the two types of capital are $c1_o^3 = \gamma^2 c1_o^1 + \gamma iy1^{1*} + iy1^{2*}$ and $c2_o^3 = \delta^2 c2_o^1 + \delta iy2^{1*} + iy2^{2*}$. We write these two optimal capital stocks as

$$c1_o^{t*} = \gamma^{t-1} c1_o^1 + \sum_{\tau<t} \delta^{t-\tau-1} iy1^{\tau*},\ t = 2, \ldots, T,$$
$$c2_o^{t*} = \delta^{t-1} c2_o^1 + \sum_{\tau<t} \gamma^{t-\tau-1} iy2^{\tau*},\ t = 2, \ldots, T. \tag{12}$$

Note that the superscripts on γ and δ are raising those variables to a power to account for depreciation, whereas the t superscripts on the other variables, A^t, $c1^t$, $c2^t$, z^t, $\tilde{z}^t$, L^t, y^t, and fy^t represent the time period.

In the static model, the output constraint for stage 2 is $y^t \leq \sum_{k=1}^{K} \lambda_{2k}^t y_k^t$. Our dynamic network program modifies stage 2 output constraint by decomposing total output, y^t, into its additive components of final consumption output and the two intermediate products. The dynamic stage 2 output constraint for t is $fy^t + iy1^t + iy2^t \leq \sum_{k=1}^{K} \lambda_{2k}^t y_k^t$.

The DEA optimization problem for DMU o is

$$\underset{\phi,\lambda_1,\lambda_2,iy1,iy2}{\text{maximize}} \sum_{t=1}^{T} \Psi^t \phi^{-1,t} \text{ subject to}$$

$$\text{in } t = 1$$

$$\begin{aligned}
&A_o^1 \geq \sum_{k=1}^{K} \lambda_{1k}^1 A_k^1, \quad c1_o^1 \geq \sum_{k=1}^{K} \lambda_{1k}^1 c1_k^1, \quad z_o^0 \geq \sum_{k=1}^{K} \lambda_{1k}^1 z_k^0, \quad \tilde{z}_o^0 \geq \sum_{k=1}^{K} \lambda_{1k}^1 \tilde{z}_k^0, \\
&z_o^1 \leq \sum_{k=1}^{K} \lambda_{1k}^1 z_k^1, \quad z_o^1 \geq \sum_{k=1}^{K} \lambda_{2k}^1 z_k^1, \quad L_o^1 \geq \sum_{k=1}^{K} \lambda_{2k}^1 L_k^1, \quad c2_o^1 \geq \sum_{k=1}^{K} \lambda_{2k}^1 c2_k^1, \\
&\phi^{-1,1} fy_o^1 + iy1^1 + iy2^1 \leq \sum_{k=1}^{K} \lambda_{2k}^1 y_k^1, \quad \lambda_{1k}^1 \geq 0, \ \lambda_{2k} \geq 0, \quad k = 1, \ldots, K
\end{aligned} \tag{13}$$

$$\begin{aligned}
&\text{in } \ t = 2, \ldots, T-1 \\
&A_o^t \geq \sum_{k=1}^{K} \lambda_{1k}^t A_k^t, \quad \gamma^{t-1} c1_o^1 + \sum_{\tau<t} \gamma^{t-\tau-1} iy1^\tau \geq \sum_{k=1}^{K} \lambda_{1k}^t c1_k^t, \quad z_o^{t-1} \geq \sum_{k=1}^{K} \lambda_{1k}^t z_k^{t-1}, \\
&\tilde{z}_o^{t-1} \geq \sum_{k=1}^{K} \lambda_{1k}^t \tilde{z}_k^{t-1}, \quad z_o^t \leq \sum_{k=1}^{K} \lambda_{1k}^t z_k^t, \quad z_o^t \geq \sum_{k=1}^{K} \lambda_{2k}^t z_k^t, \quad L_o^t \geq \sum_{k=1}^{K} \lambda_{2k}^t L_k^t, \\
&\delta^{t-1} c2_o^1 + \sum_{\tau<t} \delta^{t-\tau-1} iy2^\tau \geq \sum_{k=1}^{K} \lambda_{2k}^t c2_k^t, \quad \phi^{-1,t} fy_o^t + iy1^t + iy2^t \leq \sum_{k=1}^{K} \lambda_{2k}^t y_k^t, \\
&\lambda_{1k}^t \geq 0, \quad \lambda_{2k}^t \geq 0, \quad k = 1, \ldots, K, \ t = 2, \ldots, T-1,
\end{aligned} \tag{14}$$

$$\begin{aligned}
&\text{and in } T \\
&A_o^T \geq \sum_{k=1}^{K} \lambda_{1k} A_k^T, \quad \gamma^{T-1} c1_o^1 + \sum_{\tau<T} \gamma^{T-\tau-1} iy1^\tau \geq \sum_{k=1}^{K} \lambda_{1k}^T c1_k^T, \quad z_o^{T-1} \geq \sum_{k=1}^{K} \lambda_{1k}^T z_k^{T-1}, \\
&\tilde{z}_o^{T-1} \geq \sum_{k=1}^{K} \lambda_{1k}^T \tilde{z}_k^{T-1}, \quad z_o^T \leq \sum_{k=1}^{K} \lambda_{1k}^T z_k^T, \quad z_o^T \geq \sum_{k=1}^{K} \lambda_{2k}^T z_k^T, \quad L_o^T \geq \sum_{k=1}^{K} \lambda_{2k}^T L_k^T,
\end{aligned}$$

$$\delta^{T-1}c2_o^1 + \sum_{\tau<T} \gamma^{T-\tau-1} iy2^{\tau} \geq \sum_{k=1}^{K} \lambda_{2k}^T c2_k^T, \quad \phi^{-1,T} fy_o^T + iy1_o^T + iy2_o^T \leq \sum_{k=1}^{K} \lambda_{2k} y_k^T,$$

$$\lambda_{1k}^T \geq 0, \quad \lambda_{2k}^T \geq 0, \quad k = 1, \ldots, K. \tag{15}$$

The choice variables for the dynamic optimization problem are the expansion factors, $\phi^{-1,t}$, $t = 1, \ldots, T$, the intensity variables for the two stages of production, λ_{1k}^t and λ_{2k}^t, $k = 1, \ldots, K$ and $t = 1, \ldots, T$, and the investments in research capital and physical capital, $iy1^t$ and $iy2^t$, $t = 1, \ldots, T-1$.

New knowledge incorporated into publications is an intermediate product. Publications are produced in stage 1 and then used in stage 2 where they are combined with labor and physical capital to produce real GDP. Real GDP equals the sum of final consumption, investment in physical capital, and investment in R&D capital. The goal is to maximize the sum of the expansion factors to final consumption. Investment in physical capital and investment in R&D capital are also intermediate products; they are produced in stage 2 in t and then used in $t+1$. Physical capital investment in t adds to the depreciated stock of physical capital available to stage 2 in $t+1$. Investments in R&D add to the depreciated R&D capital stock of stage 1 in $t+1$. Therefore, it can be possible that increasing one or both types of investment in an early period can lead to expansions in subsequent periods' final consumption that more than offsets the decline from the earlier period. It can also be possible that R&D investment has a higher marginal contribution to final consumption than does physical capital investment. Or *vice versa*. The goal of our model is to find out and see if it coincides with Romer's theory.

4. Data and model estimates

To estimate the dynamic network model, we use pooled aggregate data on 53 countries for the years 1996–2012. The process of creating new knowledge generally occurs over more than one year as ideas are first discussed and refined, then experiments are performed and analyzed, with results presented at conferences, and finally, formalized as publications. We follow Weber and Xia (2011) and Fukuyama et al. (2016) and take a three-year moving average of authors, publications, and citations. The three-year averages along with lagged investments in physical capital and knowledge capital allow the model to be estimated for 1999–2012. Most of the input and intermediate product data for the first stage come from Bostian et al. (2018) and include counts of authors, publications, and highly cited publications for each of 18 STEM disciplines aggregated to the country level for the years 1996–2012.[2] The knowledge spillover received by country j equals the sum of publications across all countries minus country j's publications.

Research capital is derived from the World Bank, which reports the percentage of real GDP allocated to R&D. We multiply this percentage by real GDP at purchasing power parity in constant (2015) dollars. Since research capital investments are not reported for some countries in some years, we interpolate missing values. Then, we take the three-year moving average of R&D expenditures as the measure of research capital.

[2]The STEM disciplines from the Scopus ASJC main discipline codes are AGRI, BIOC, CENG, CHEM, COMP, DECI, EART, ECON, ENER, ENGI, ENVI, IMMU, MATE, MATH, MEDI, NEUR, PHAR, PHYS.

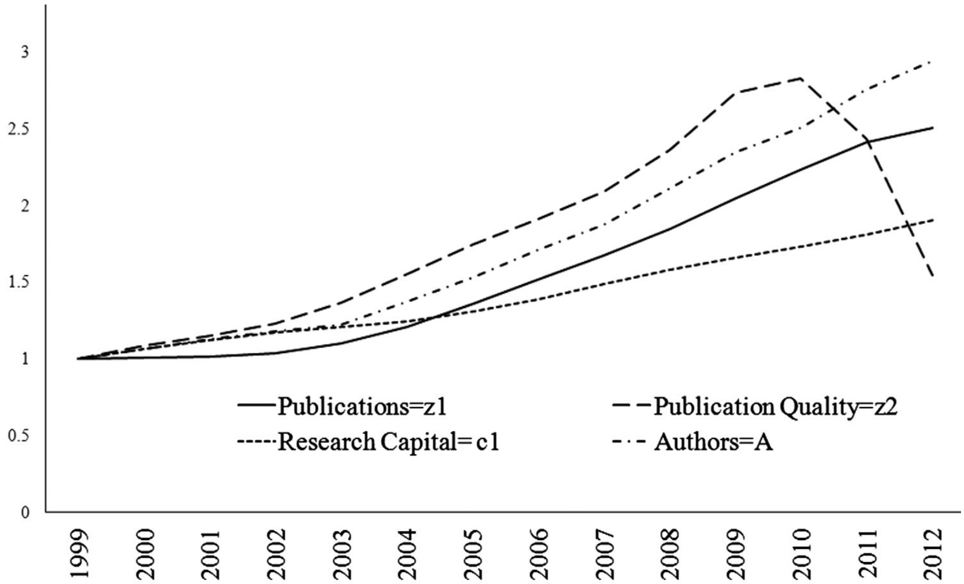

Fig. 3. Trend in knowledge inputs and outputs in 18 STEM disciplines.

The intermediate products from stage 1 (publications—z_1 and citations—z_2) enter stage 2 technology as inputs that are combined with labor and real physical capital to produce real GDP. These stage 2 inputs and outputs come from the Penn World Tables (see Feenstra et al., 2015). New publications from t spill over to other countries in $t + 1$. Our one-period lag on spillover publications is consistent with Luintal and Khan's (2017) review that international knowledge spillovers occur quickly.

Table 2 reports descriptive statistics for the pooled data and for the beginning and ending sample years, 1999 and 2012. The USA has the maximum real GDP and China the maximum employment. Average real physical capital ($c2$) exceeds the three-year average of R&D expenditures ($c1$) by a factor of 42. The average percentage of real GDP allocated to R&D expenditures was 1.32%. On average, authors exceed the number of publications. We measure publication quality by the number of citations in the four years after the publications occur.[3] Our choice to measure citations in the four years after publication is near the midpoint of recent work by Parolo et al. (2015) who found that citations in physics, medicine, chemistry, and biology peak between two and seven years after publication. From 1999 to 2012, average real physical capital doubles and knowledge capital increases by 90%. Average employment increases 15% and real final consumption increases 68%.

Figure 3 graphs the trend in average publications, authors, and citations. The variables are normalized to 1 in 1999. From 1999 to 2012 average R&D expenditures increase by a factor of 1.9, authors increase by a factor of 2.9, and publications increase by a factor of 2.5. However, citations increased only 1.5 times and peaked in 2010 and then declined in 2011 and 2012. This decline in citations is due to an incomplete updating and assessment of citations, that is, citations to

[3]For every additional period used to measure citations, like five years instead of four, the available sample years to run the model is reduced by 1.

Table 2
Descriptive statistics, pooled data, 742 observations

Variables	Model	Description	Mean	SD	Min.	Max.
Pooled data, 742 observations = 53 countries × 14 years						
y	S,D	Real GDP	1,218,000	2,395,000	40,000	15,900,000
L	S,D	Employment	40	120	1	790
$c2$	S,D	Real physical capital	3,844,000	7,550,000	66,000	54,610,000
$c1$	S,D	Knowledge capital	61,000	168,000	400	1,280,000
z_1	S,D	Publications	42,700	89,100	860	707,000
z_2	S,D	Citations	275,000	675,000	2100	6,300,000
$\tilde{z}^{t-1}$	S,D	Spillover publications	1,918,000	651,000	537,000	3,280,000
z_1^{t-1}	S,D	Past own publications	40,000	84,000	770	701,000
A	S,D	Authors	88,000	190,000	1600	1,530,000
$fy = y - iy1 - iy2$	D	Real final consumption	880,000	1,660,000	27,000	12,160,000
$iy1$	D	R&D investment	22,000	58,100	200	400,000
$iy2$	D	Physical capital investment	320,000	730,000	5000	6,940,000
1999						
y	S,D	Real GDP	90,9000	1,890,000	40,000	12,510,000
L	S,D	Employment	40	110	1	700
$c2$	S,D	Real physical capital	2,600,000	5,140,000	66,000	34,400,000
$c1$	S,D	Knowlecdge capital	44,000	132,000	400	899,000
z_1	S,D	Publications	27,000	61,000	860	413,000
z_2	S,D	Citations	154,000	430,000	2100	3,000,000
$\tilde{z}^{t-1}$	S,D	Spillover publications	930,000	63,000	537,000	960,000
z_1^{t-1}	S,D	Past own publications	27,000	63,000	770	424,000
A	S,D	Authors	50,000	110,000	1600	720,000
$fy = y - iy1 - iy2$	D	Real final consumption	675,000	1,350,000	27,000	8,910,000
$iy1$	D	R&D investment	15,000	47,000	200	318,000
$iy2$	D	Physical capital investment	219,000	496,000	5000	3,290,000
2012						
y	S,D	Real GDP	1,590,000	3,019,442	54,200	15,900,000
L	S,D	Employment	44	126	1	790
$c2$	S,D	Real physical capital	5,650,000	10,360,000	297,000	54,610,000
$c1$	S,D	Knowledge capital	83,000	207,000	1900	1,280,000
z_1	S,D	Publications	68,000	128,000	5000	707,000
z_2	S,D	Citations	238,000	484,000	8800	3,200,000
$\tilde{z}^{t-1}$	S,D	Spillover publications	3,220,000	120,000	2,580,000	3,280,000
z_1^{t-1}	S,D	Past own publications	66,000	125,000	4670	701,000
A	S,D	Authors	150,000	290,000	8000	1,530,000
$fy = y - iy1 - iy2$	D	Real final consumption	1,130,000	2,020,000	39,800	12,160,000
$iy1$	D	R&D investment	29,000	712,000	666	400,000
$iy2$	D	Physical capital investment	434,000	1,050,000	13,000	6,950,000

Real GDP means real physical capital and knowledge capital are millions of constant 2015 dollars. Employment is millions of workers.
S, D means variable is in static and dynamic model; D means variable is in dynamic model.

Table 3
Static estimates of stage 1 and network efficiency

	With citations	Without citations	
Year	Stage 1	Stage 1	Network
1999	0.975	0.974	0.823
2000	0.982	0.979	0.800
2001	0.974	0.967	0.794
2002	0.979	0.971	0.783
2003	0.975	0.972	0.769
2004	0.973	0.971	0.766
2005	0.969	0.967	0.750
2006	0.972	0.970	0.727
2007	0.971	0.967	0.739
2008	0.973	0.969	0.682
2009	0.965	0.961	0.753
2010	0.968	0.963	0.732
2011	0.961	0.958	0.694
2012	0.964	0.958	0.695

publications that occurred in 2011 and 2012 were not updated in the last years of our sample, 2015 and 2016.

We estimate static efficiency by country and year for stage 1 two ways: with two intermediate products of publications and citations and with a single intermediate product of publications. The efficiency estimates are in Table 3. By construction, average efficiency is always greater when controlling for citations than when citations are not included. Nonetheless, output technical efficiency ranges between 0.96 and 0.98 and the estimates with and without citations are close in value. In 2001, the two efficiency estimates have a Spearman rank correlation coefficient of 0.85. In the other years, the Spearman rank correlation coefficients are greater than 0.93. Furthermore, the citations to published work from 2011 and 2012 were not completely updated and assessed in 2015 and 2016. Therefore, given the potential for bias in citations, and the significantly high correlation between the two models, we opt for a more parsimonious model and do not include citations as a quality control in the dynamic model estimates presented below.

Average static network efficiency is 0.82 in 1999, but then trends down to 0.68 in 2008. Average efficiency rebounds to 0.75 in 2009, but declines to 0.69 in 2011 and 2012.

We estimate the dynamic model for a three-year horizon assuming equal weights for each period: $\Psi^t = 1$.[4] We experimented with several rates of depreciation for knowledge capital and physical capital with qualitatively similar results. We settled on a depreciation rate for knowledge capital of $1 - \gamma = 0.33$ and a depreciation rate for physical capital of $1 - \delta = 0.10$. Dynamic efficiency, EFF, equals the average sum of the reciprocals of the expansion factors, that is, $EFF = \frac{1}{3}(\frac{1}{\phi^t} + \frac{1}{\phi^{t+1}} + \frac{1}{\phi^{t+2}})$. The dynamic estimates are presented in Table 4.

Table 4 compares the dynamic capital (DC) estimates from (13), (14), and (15) with the static network estimates presented in Table 3. Since the status quo amounts of research capital to stage 1

[4]We also estimated the model using discount rates of $r = 0.05$ and $r = 0.10$ and obtained similar estimates.

Table 4
Efficiency estimates

Years	Dynamic capital (DC)	Dynamic investment (DI)	Capital preservation (CP)
1999–2001	0.673	0.687	0.702
2000–2002	0.670	0.666	0.718
2001–2003	0.687	0.633	0.739
2002–2004	0.692	0.622	0.776
2003–2005	0.711	0.633	0.799
2004–2006	0.729	0.642	0.792
2005–2007	0.721	0.594	0.777
2006–2008	0.691	0.604	0.708
2007–2009	0.679	0.645	0.730
2008–2010	0.677	0.676	0.749
2009–2011	0.708	0.705	0.769
2010–2012	0.701	0.642	0.746
All years	0.695	0.646	0.751

and physical capital to stage 2 are feasible, but not necessarily optimal in the dynamic model, average efficiency in the dynamic model should be no greater than the three-year average efficiency for the static model. For example, average dynamic efficiency is 0.673 during 1999–2001, while average static efficiency for those same three years is $(0.823 + 0.800 + 0.794)/3 = 0.806$. Thus, an optimal reallocation of research capital and physical capital across three years expands the technology set allowing greater potential output. Average efficiency for the DC model ranges from 0.67 during 2000–2002 to 0.729 during 2004–2006.

In a precursor to this paper, two alternative specifications of research capital and physical capital were considered. The "dynamic investment" (DI) model follows Färe et al. (2018). In this specification, we drop the constraints for the stocks of research capital and physical capital and replace them with their respective flows of research investments and physical capital investments. Our "capital preservation" (CP) model follows Nemoto and Goto (2003), Emrouznejad and Thanassoulis (2005), and Ouellette and Yan (2008). In this specification, we take the stocks of research capital and physical capital as values to be preserved for future use.[5] Dynamic efficiencies for these two alternative specifications are reported in Table 4. Efficiency averages 0.751 for the CP model, versus 0.695 for the DC model and 0.646 for the DI model. The CP model has the highest average efficiency in every three-year period. In contrast, the DI model estimates the lowest average efficiency in every three-year period except 1999–2001. The DC and CP models show greater average efficiency in 2010–2012 than in 1999–2001. In contrast, the DI model shows a downward trend in average efficiency. The efficiency estimates for the three models exhibit positive significant correlation coefficients between the three models in every year: between 0.78 and 0.91 for the DC and CP models; between 0.55 and 0.77 for the DC and DI models; and between 0.70 and 0.87 for the DI and CP models.

We are further interested in examining Romer's theory that there will be underinvestment in R&D capital relative to physical capital because of the public good characteristics of knowledge. By construction, the CP model ensures that the optimal investments in physical capital and research capital are great enough to preserve the two capital stocks to be at least as great as their actual levels

[5]Details of the DI model and the CP model are found in Bostian et al. (2018).

Table 5
Terminal values of actual and optimal research capital and physical capital

	Actual		Optimal			
Years	$c1^T$	$c2^T$	$c1^{T*}$	$c2^{T*}$	$\frac{c1^T}{c2^T}$	$\frac{c1^{T*}}{c2^{T*}}$
1999–2001	49,100	2,740,000	39,700	1,240,000	0.012	0.020
2000–2002	51,200	2,790,000	40,600	1,200,000	0.013	0.020
2001–2003	52,800	2,930,000	43,900	1,240,000	0.013	0.021
2002–2004	54400	3,180,000	43,200	1,360,000	0.012	0.020
2003–2005	57,300	3,500,000	44,400	1,490,000	0.012	0.018
2004–2006	60,700	3,900,000	47,100	1,650,000	0.011	0.018
2005–2007	65,000	4,250,000	49,700	1,820,000	0.011	0.016
2006–2008	69,200	4,540,000	52,200	2,120,000	0.011	0.017
2007–2009	72,600	4,730,000	55,400	2,400,000	0.011	0.017
2008–2010	75,600	4,980,000	58,000	2,460,000	0.011	0.018
2009–2011	79000	5,350,000	60,300	1,970,000	0.011	0.020
2010–2012	83,100	5,650,000	62,400	2,100,000	0.011	0.019
All years	64,200	4,040,000	49,000	1,750,000	0.012	0.018

in the terminal period, so this model does not shed any light on Romer's theory. In turn, the two stocks of capital are omitted from the DI model, as this model includes only the flows of investment in R&D and physical capital. Given the high correlations between these models, we focus on the DC model for the rest of this paper.

The optimal amounts of research capital and physical capital in $T = 3$ (the terminal period) are calculated as

$$\begin{aligned} c1_k^{3*} &= \gamma^2 c1_k^1 + \gamma iy1^{1*} + iy1^{2*} \\ \text{and } c2_k^{*3} &= \delta^2 c2^1 + \delta iy2^{1*} + iy2^{2*}. \end{aligned} \tag{16}$$

Table 5 shows that on average, both types of capital would shrink relative to their actual values as more resources are pushed into producing final consumption. However, their relative shares would also change as shown in the last two columns. The optimal ratio of research to physical capital is greater than the actual ratio of research to physical capital in every year. The results are even more pronounced in some countries such as the United States where optimal research capital investments are greater than actual research capital investments in every year. In addition, the optimal physical capital stock in the United States would shrink relative to its actual level. In the United States, the increase in research capital and decrease in physical capital relative to actual levels would result in an optimal research to physical capital ratio almost twice as great as the actual ratio. For instance, during 1999–2001, the terminal period ratio of actual knowledge capital to actual physical capital is 0.027 but the ratio of optimal knowledge capital to optimal physical capital is 0.042. In our last three-year period, 2010–2012, the actual ratio is 0.026, but the optimal ratio is 0.053. In relative terms then, our empirical results are consistent with Romer's (1990) theory that predicts underinvestment in research capital because of its spillover effects and because prices in stage 2 are greater than the competitive level.

Table 6
Actual and optimal ratio of research capital ($c1$) to physical capital ($c2$) for six countries with optimal investment in research capital

Country	Actual ratio $c1^T/c2^T$	Optimal ratio $c1^{*T}/c2^{*T}$
China	0.013	0.039
Iran	0.005	0.010
New Zealand	0.012	0.024
Saudi Arabia	0.002	0.003
United Kingdom	0.015	0.030
United States	0.025	0.048
Six country mean	0.012	0.026
Mean for other 47 countries	0.012	0.017

Six countries (China, Iran, New Zealand, Saudi Arabia, United Kingdom, and United States) have optimal R&D investments that are equal to actual R&D investments in every three-year period 1999–2001 and 2010–2012, but these same countries overinvested in physical capital relative to knowledge capital. Table 6 reports means of the actual and optimal ratios of R&D capital (c_1) to physical capital (c_2) in the terminal period ($T = 3$) and compares them to the mean ratios for the other 47 countries in the sample. The optimal amount of physical capital in the terminal period is smaller than the actual quantity for the six countries. Thus, the final outputs could be produced using the same amount of R&D capital and labor, but less physical capital. Therefore, the excess physical capital gets pushed into the final output via the expansion factors λ^t. In addition, the six countries account for 48–50% of actual cumulative R&D spending among all 53 countries, and the two biggest economies, United States and China, account for 44–46% of all R&D spending. These results point to an overallocation of resources to physical capital. In addition, the work by Peri et al. (2015) found that 30–50% of productivity growth in U.S. cities was due to the growth of foreign STEM researchers. Our model uses the number of authors in a country as a fixed input in the research sector, but does not consider reallocations of STEM researchers across countries. Allowing reallocation of researchers between countries might be another way of increasing final output, but this reallocation is not part of our model and in practice is restricted by governments' immigration policies.

Table 7 reports the ratios by year for the six countries reported earlier and for the other 47 countries. For the six research-efficient countries the actual ratios of R&D to physical capital, average 1.2%, but with optimal ratios that range from 1.9% in 2007–2009 to 3.2% in 2010–2012. The other 47 countries have similar actual ratios of R&D to physical capital, about 1.2%, but the optimal terminal period ratios of R&D to physical capital range only from about 1.5% to almost 2%. Thus, it appears that the six research-efficient countries, especially China and the United States should allocate more to R&D capital relative to physical capital.

5. Conclusions

The transmission of existing knowledge and the production of new knowledge are important drivers of productivity growth. We construct a dynamic network technology where knowledge is an

Table 7
Ratios of research capital to knowledge capital by year

Period	6 Countries		47 Countries	
	Actual ratio $c1^T/c2^T$	Optimal ratio $c1^{*T}/c2^{*T}$	Actual ratio $c1^T/c2^T$	Optimal ratio $c1^{*T}/c2^{*T}$
1999–2001	0.012	0.025	0.012	0.019
2000–2002	0.013	0.027	0.013	0.019
2001–2003	0.013	0.027	0.013	0.020
2002–2004	0.012	0.027	0.012	0.019
2003–2005	0.012	0.026	0.012	0.017
2004–2006	0.012	0.025	0.011	0.017
2005–2007	0.012	0.025	0.011	0.015
2006–2008	0.012	0.023	0.011	0.017
2007–2009	0.012	0.019	0.011	0.016
2008–2010	0.012	0.021	0.011	0.017
2009–2011	0.012	0.032	0.011	0.018
2010–2012	0.012	0.032	0.011	0.017

intermediate product. The network model consists of two stages where knowledge, measured as the number of scientific publications in STEM fields, is produced in stage 1 and then used as an input in stage 2 where it is combined with labor and physical capital to produce final consumption and two types of investment spending: investment in new research capital and investment in new physical capital. A country experiences dynamic effects in several ways. First, the two types of investment spending that occur in period t augment the country's production possibilities in $t + 1$. Second, new knowledge (publications) produced by a country in t spills over and augments the country's production in period $t + 1$. Third, although new knowledge produced by a country in t can only be used by the same country in t, by period $t + 1$ that same knowledge spills over to other countries as it becomes more widely disseminated.

We use panel data on 53 countries during 1999–2012 to estimate the model using DEA. Dynamic efficiency averages about 70% compared with the average static efficiency of about 75%. Thus, outputs could be increased the most by countries realizing greater technical efficiency in converting publications and other inputs into real GDP. But there are still some gains that could be realized by an intertemporal reallocation of the investments in research capital and physical capital. For the 53 countries, the average ratio of research capital to physical capital ranges is about 0.012 and the optimal ratio of research capital to physical capital is 0.019. This result is consistent with Romer's (1990) theory of endogenous technical change, which showed that private firms will tend to underallocate resources to research because of the public good properties of knowledge.

Our dynamic optimization problem takes a country's research capital as given in the initial period and that capital depreciates over time. Each subsequent period's capital equals depreciated capital plus new research investment. A country is research efficient when the optimal amount of research capital (depreciated capital plus new investment) equals the actual amount of research capital and the country produces on the frontier of stage 1 technology. Six countries—China, United States, United Kingdom, Iran, Saudia Arabia, and New Zealand—produced on the research frontier every year. For the six research-efficient countries, the ratio of optimal R&D capital to optimal physical capital increases from around 0.025 in 1999–2001 to 0.032 in 2010–2012. The same ratio trends downward for the other 47 countries, from 0.019 to 0.0165. China and the United States accounted for 48–50% of cumulative R&D spending among the 53 countries and these two countries have the highest average ratio of optimal R&D to optimal physical capital, 3.9% and 4.8%, respectively.

In 2000, European countries set an ambitious goal of allocating 3% of GDP to R&D by 2010 in what is known as the "Lisbon strategy" (Bongardt and Torres, 2013). Only a few countries in our sample met this goal by 2010: Denmark, Finland, Sweden, and Switzerland in Europe, along with Korea, Japan, and Israel. Our results indicate that increasing R&D investments relative to capital investments can increase potential outputs. But we caution that those resources would have to be used efficiently to reach potential. Since our model estimates that countries are on average only 70% efficient, some of that potential output will likely disappear as various inefficiencies arise. Nonetheless, countries such as China, Korea, and Iran have increasingly allocated resources to R&D and have also realized relatively strong economic growth (Zhou and Leydesdorf, 2006; Moed and Galevi, 2014; Moed, 2016).

We offer several caveats of our study. First, the data are highly aggregated, which masks most of the interactions between the actual researchers within and across countries and disciplines. Second, our model might suffer from a "curse of dimensionality" as the relatively small number of countries, and a large number of outputs and inputs probably puts an upward bias to the efficiency estimates. Daraio and Simar (2007) propose a factorial analysis to reduce this curse, and future research might usefully incorporate factor analysis into the dynamic network model. Third, our model has used only publications as the knowledge output. Private R&D divisions of businesses and increasingly, universities, also produce patents, which is another knowledge output. Future research might usefully account for these patents in a dynamic network model.

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