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S.I.: RECENT DEVELOPMENTS IN FINANCIAL MODELING AND RISK MANAGEMENT

Atheoretical Regression Trees for classifying risky financial institutions

Carmela Cappelli[1] · Francesca Di Iorio[1] · Angela Maddaloni[2] · Pierpaolo D'Urso[3]

Abstract
We propose a recursive partitioning approach to identify groups of risky financial institutions using a synthetic indicator built on the information arising from a sample of pooled systemic risk measures. The composition and amplitude of the risky groups change over time, emphasizing the periods of high systemic risk stress. We also calculate the probability that a financial institution can change risk group over the next month and show that a firm belonging to the lowest or highest risk group has in general a high probability to remain in that group.

Keywords Systemic risk · Financial stress · Atheoretical Regression Trees · Factor analysis

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✉ Carmela Cappelli
carcappe@unina.it

Francesca Di Iorio
fdiiorio@unina.it

Angela Maddaloni
angela.maddaloni@ecb.europa.eu

Pierpaolo D'Urso
pierpaolo.durso@uniroma1.it

1 Dipartimento di Scienze Politiche, Università Federico II di Napoli, Via L. Rodinò n. 22, 80138 Naples, Italy

2 European Central Bank, Sonnemannstrasse 20, 60314 Frankfurt am Main, Germany

3 Dipartimento di Scienze Sociali ed Economiche, Sapienza Università di Roma, P.le Aldo Moro 5, 00185 Rome, Italy

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1 Introduction

The recent great financial crisis has brought to the center stage the issue of evaluating the systemic risk of the financial system, possibly using techniques that would allow to identify pockets of accumulation of risk.

Individual financial institutions use several approaches to measure and manage their risk. Some of these approaches have been enshrined in prudential regulations, and therefore are commonly used by supervisors and financial institutions alike. The Value-at-Risk (*VaR*), for example, is the most widely-used risk measure by financial institutions, and it has been used by regulators and policy makers to determine "capital levels that need to be set aside by financial firms against market risks". However the *VaR*, and generally all measures aimed at evaluating the risk of a financial entity in isolation, falls short in capturing the overall risk accumulating in the system.

Many different approaches to address this issue have been proposed in recent years and several alternative indicators of risk measures are used by policy institutions and researchers. A comprehensive survey of the empirical literature on systemic risk measurement can be find for example in ECB (2009), ECB (2010) and ECB (2011). A widely known risk indicator is for example the Conditional Value-at-Risk (*CoVaR*), defined as the *VaR* of institution i conditional on institution j being in financial distress, where institution j is at its *VaR*. This measure has also been refined by using the ΔCoVaR (Adrian and Brunnermeier 2016). A number of largely known indicators are based on market capitalization values, like the systemic risk measure (*SRisk*) that captures the expected capital shortage of a firm given its degree of leverage and the Marginal Expected Shortfall (*MES*), see Brownlees and Engle (2017). Other indicators are the MES of a financial firm, introduced by Acharya et al. (2017) as the average return of each firm during the 5% worst days for the market, and the so-called "CAPM beta times market capitalization" (Benoit et al. 2013). Other measures draw from balance sheets items, like the Leverage Ratio (Fostel and Geanakoplos 2008; Geanakoplos and Pedersen 2012).

All these measures have drawbacks which of course may limit their performance and reliability. The main issue is choosing a threshold over which the risk of financial institutions is considered worrisome. A simple approach is to rank the institutions according to the considered risk indicators, for example focussing on the top 10 riskier financial firms, i.e. those showing the highest (worst) values of the indicators [see for example Acharya et al. (2012)].This method is undoubtedly easy to implement and allows a quick comparisons across different risk indicators and thus it is particularly useful for practitioners that routinely analyze systemic risk index time series. However it doesn't take into account the concentration of risk in the system and the relative riskiness of financial institutions. For example, in terms of value of the indicator, the 20th firm may be equally risky than the first, or the risk gap between the first and the third could instead be significant. Second, typically different indicators produce different rankings and it is difficult to choose a priori one over the others. Therefore, researchers and policy makers alike have been drawn to ask the following question: is there a way to summarize the different information arising from the indicators and obtain an effective and quick judgment on the riskiness of financial firms? Several authors have proposed different approaches based on the principal component analysis (PCA), e.g. Billio et al. (2012) and Rodríguez-Moreno and Peña (2013). In more recent contributions, Giglio et al. (2016), use conditional quantile factor model to obtain systemic risk indices. The work more closely related to ours is Nucera et al. (2016). They apply the factor analysis across rankings in the cross-section dimension, at each point in time, to form optimal combinations of rankings.

We contribute to this literature by proposing a novel approach based on a combination of factor analysis and a binary recursive partitioning approach known as Atheoretical Regression Trees (ART for short) originally proposed by Cappelli and Reale (2005) to detect multiple change points in time series as in Cappelli et al. (2008) and Rea et al. (2010) and then successfully extended to fuzzy time ordered units and to interval value time series (Cappelli et al. 2013, 2015).

Several risk measures are summarized using PCA and then the ART procedure is applied to the ordered principal component scores. In this way, a partition of the synthetic index and, accordingly, a classification of financial firms into homogeneous risk groups are defined. This approach has the advantage of clearly identifying, at each point in time, the riskiest institutions, independently of their rankings. The number of financial institutions belonging to each group provides also a clear indication of where the risk are mounting in the system. In particular, we find that the amplitude of the groups changes over time and identify periods of high systemic risk, when a larger number of institutions belong to the riskiest group. This further support our claim that policy makers and supervisor should not restrict their attention to a predefined number of financial institutions.

Next, we analyze the probability of transition from one group to another. We generally find that the lowest and the highest risk group are like “absorbing states”. Once an institution is part of one of these groups, it is very likely that it will remain in that group. This has a number of implications for supervisors and for financial stability policy makers. It may guide them to choose on which institutions focus their monitoring efforts.

The paper is structured as follows. In Sect. 2 we provide the necessary background on regression tree methodology and ART. Section 3 outlines the proposed approach to classify financial institutions based on the composite risk indicator. Section 4 illustrates the risk measures and the data used in the analysis. Section 5 reports and discusses the results. Some concluding remarks follow in Sect. 6.

2 Background

This section provides some details on both regression tree methodology and the ART procedure which is based on Least Square Regression Trees (LSRT for short).

LSRT express the relationship between a response variable and a set of covariates in the form of a binary tree in which every node i.e. a group of observations, is split into two subgroups, the left and the right child nodes (subsets of observations), respectively. LSRT can be seen as piecewise constant regression models as they partition the covariate space into regions (the tree nodes) and fit a constant value within each region that is the mean of response values in the given node.

The top-down partitioning algorithm employs a splitting criterion to choose at each tree node the best split, i.e. the binary division, of the current node.

Specifically, given a numerical response variable Y and a set of covariates $(X_1, X_2, \ldots, X_p)$ observed on a sample of N units, the binary tree arises by recursively splitting the training set $(y_i, x_{i1}, \ldots, x_{ij}, \ldots, x_{ip})_{i=1}^{N}$, into two subsets seeking at any internal node h for the split s that breaks the nodes into two mutually exclusive subsets as homogeneous as possible with respect to the given response variable (for details and a review on tree based methods see Breiman et al. (1984) and Loh (2011) respectively). Thus, the best split provides the highest reduction in deviance

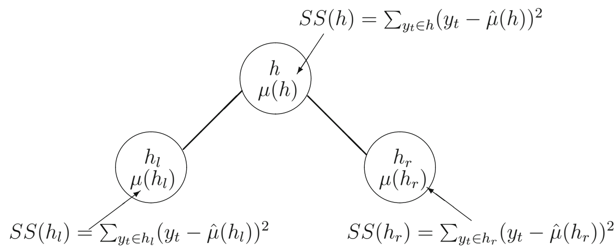


Fig. 1 Example of a split in a binary regression tree

$$\Delta SS(h, t) = SS(h) - [SS(h_l) + SS(h_r)] \tag{1}$$

where $SS(h) = \sum_{y_i \in h}(y_i - \hat{\mu}(h))$ is the sum of squares for node h, and $SS(h_l)$ and $SS(h_r)$ are the sums of squares for the left and right descendants, respectively; an example of a split in a regression tree is displayed in Fig. 1.

As h_l and h_r are an exhaustive partition of h, $SS(h)$ represents the total sum of squares $TSS_y(h)$ and $SS(h_l) + SS(h_r)$ the within-group sum of squares $WSS_{y|s}(h)$. Therefore the splitting criterion (1) is equivalent to maximize the between-groups sum of squares $BSS_{y|s}(h)$ that can be written as:

$$BSS_{y|s}(h) = \frac{N(h_l)N(h_r)}{N(h)^2}(\hat{\mu}(h_l) - \hat{\mu}(h_r))^2 \tag{2}$$

where $N(h)$ denotes the number of Y' values in node h and $N(h_l)$ and $N(h_r)$ the corresponding subsets that go to left and right child nodes, respectively. Thus, in LSRT the splitting criterion searches for the child nodes that are as far away as possible, i.e., the two subgroups of Y's values for which the squared distance between the corresponding means is maximum. Once a node is partitioned, the splitting process is recursively applied to each child node until either they reach a minimum size or no reduction of the node deviance can be achieved.

Minimizing the within-group sum of squares (or, equivalently, maximizing the between group sum of squares) is a natural clustering criterion for grouping a single real variable (Everitt et al. 2001). This is the case of the Fisher's algorithm of exact optimization (Fisher 1958) that introduces the notion of *contiguous partitions*. Let i, i' and i'' be three data points having assigned a numerical measure such that $Y_i < Y_i' < Y_{i''}$; according to Fisher a partition is said to be *contiguous* if it consists of groups that satisfy the following condition: if i and i' are assigned to the same class then i'' must be also assigned to that class.

Fisher demonstrates that least square partitions are contiguous and he provides a dynamic programming approach that allows to find the exact optimal partition into G groups drastically reducing the number of computations.

ART exploits the concept of contiguous partitions within the framework of LSRT using as a single covariate an arbitrary sequence of completely ordered numbers $K = 1, 2, \ldots, i, \ldots, N$. Tree-regressing the response variable Y on this artificial covariate resorts to create and check at any node h all possible binary contiguous partitions of the $Y_i \in h$. These splits are the only ones that need to be checked to detect the binary partition that minimizes the sum of squares and, indeed, they are generated by using K as covariate. In other words, for the contiguity property the best split lays in K (or in its subintervals after the split of the root note has taken place) and the tree algorithm, based on the splitting criterion in the (1), is forced to identify it. In general, the use of K as covariate enables ART to generate G contiguous groups such that the mean of each group satisfies $\hat{\mu}_g \neq \hat{\mu}_{g+1}$ with $g = 1, \ldots, G$ and $\sum_g N_g = N$.

It's worth noting that, within ART, the target variable Y is partitioned preserving its internal sorting; for example, if the target variable is a time series, the partition will consist of groups of observations that retain the temporal ordering and for this reason the procedure has been successfully employed to locate multiple changes occurring at unknown dates in various types of time series (Cappelli and Di Iorio 2010; Cappelli et al. 2013, 2015).

Eventually, note that as K is not a real predictor variable but an artificial covariate and the procedure is theory-free, it has been called Atheorethical Regression Trees.

3 Proposed approach for classification of financial risky institutions

As described in the introduction our aim is to obtain a classification of financial firms into homogeneous risk groups using a summary measure of risk. Our procedure is composed by the following three main steps which are repeated for every month in our dataset:

- Step 1: Summarize the systemic risk measures in a single composite index using PCA
- Step 2: Apply the Atheoretical Regression Trees procedure (ART) on the ordered individual scores given by the first principal component (PC) to obtain a partition of the synthetic index into contiguous classes that identifies risk groups.
- Step 3: Apply pruning strategy to the resulting tree to select the optimal partition.

Specifically, we perform the PCA on the monthly correlation matrix of the original numerical values. In doing so we depart from Nucera et al. (2016). Indeed for each indicator they turn the observed data into ranks for the selected units and then they perform the PCA on the resulting transformed data. Our approach has the advantage to avoid loss of information. In particular preserving the numerical nature of the original variables is useful for the detection of outliers which are quite common in financial data.

We detect and treat the outliers using the following procedure. First we run PCA on the whole monthly data set and then we identify as outliers the observations that lay beyond the 95% whiskers of the box-plot of PC scores, on both sides. Then the PCA is performed again on the restricted sample excluding the outliers. The outliers are treated as supplementary individuals and their coordinates are computed using the PCA performed on the subset of active individuals. In this way we obtain the scores for the remaining individuals net of the outliers effect; at the same time we keep them in the analysis. As an example, in Fig. 2 we report for November 2015 the effect of 4 evident outliers on the first PC scores. To help the cross monthly comparison we rescale the PC individual scores in $[-1; 1]$.

The ordered individual scores obtained by the first PC are the realizations of a new continuous variable that can be partitioned into classes of increasing systemic risk using the ART method.

Thus, since in our application the variable to be partitioned is given by the first principal component scores sorted in ascending order, ART generates a partition such that $\hat{\mu}_g < \hat{\mu}_{g+1}$ while the split points identify the thresholds that separates classes of increasing risk. By partitioning the ordered composite systemic risk index by maximum homogeneity, ART provides a classification of the financial institutions that can be employed for decision purposes, i.e. to predict the class of risk associated to a new company simply computing its PC score and then checking to which group (terminal node) it belongs.

It's worth noting that the recursive partitioning algorithm tends to create a large tree that overfits the data. In the present context overfitting leads to a partition into an overly

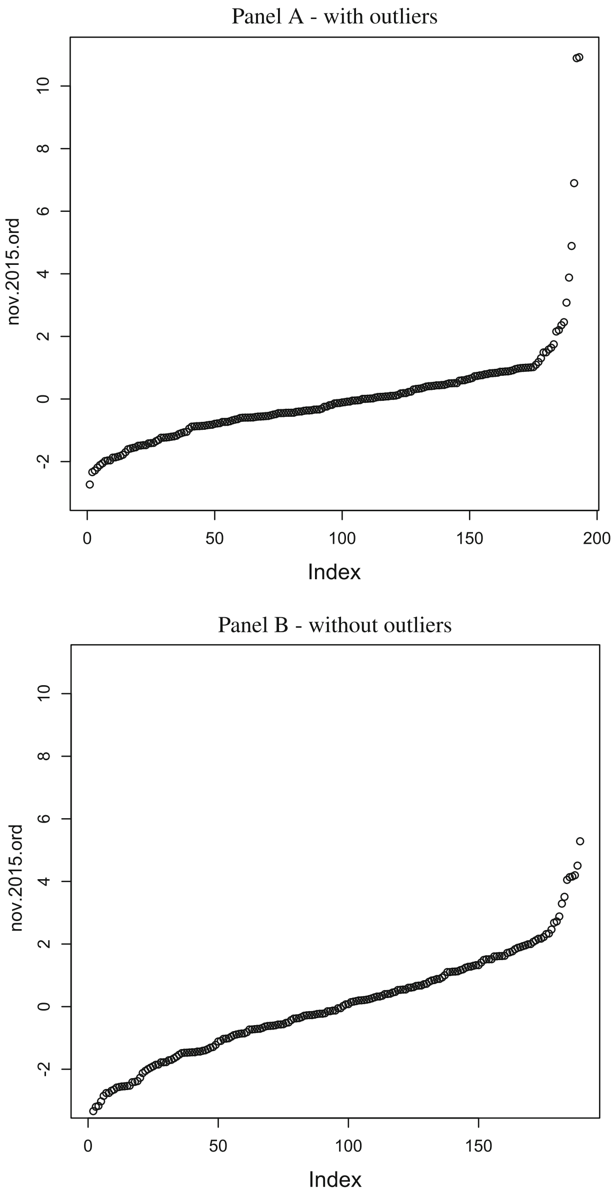


Fig. 2 Outliers effects on 1st PC scores, November 2015

large number of groups and we need to find a parsimonious subtree whose terminal nodes correspond to the actual number of distinct subgroups present in the data. This is a major issue in recursive partitioning methods. Several solutions have been suggested in the literature including rules of thumb, informal methods, statistical tests as discussed in Cappelli et al. (2002). In the present case we have considered the classical pruning method proposed in the framework of the CART approach (Breiman et al. 1984) that selects the subtree (partition) that produces the smallest cross validated error.

4 Data

We use monthly observations of the six systemic risk indicators calculated for $N = 193$ financial sector European firms.[1] Our sample contains commercial banks, private and investment banks, insurance and asset managers based in Europe. Of these, 23% are UK based, 10% are in France, Germany and in Italy, and 8% in Switzerland. Commercial banks represent 44% of the sample, around 36% are investment banks, while the remaining 20% is composed of asset managers, financial providers and the insurances. We restrict the sample period to January 2010–December 2015 ($T = 72$ months), in order to obtain a balanced panel and cover the main European debt crisis period.

Following the work by Nucera et al. (2016) we consider six commonly used measures of systemic risk. The risk measures: *SRisk*, Marginal Expected Shortfall (*MES*), Leverage ratio (Lvg), and Dollar systematic risk ($\beta \times$ MV) are from the $vLab$ website.[2] The $\Delta CoVaR$, and Value-at-Risk (*VaR*) are taken from the confidential Bundesbank submissions to the European Systemic Risk Board (ESRB).[3] As pointed out by Nucera et al. (2016) these risk measures represent a comprehensive set of market based measures that regulators may take into account in practice.

The risk measures that we consider are derived following different procedures and they take into account different dimension of the riskiness of the financial institutions involved. We provide below a short description of all the measures, we refer to Nucera et al. (2016) for more details and to the *vLab* site for technical aspects.

1. *SRisk* is an estimate of the capital shortfall a given financial firm is expected to experience conditional on a severe market decline, see Brownlees and Engle (2017), and can be interpreted as the systemic risk contribution of a given financial firm. *SRisk* is a function of a firm's size, leverage, and its expected equity loss given a market downturn.
2. *MES* introduced by Acharya et al. (2017) is evaluated in the version proposed by Brownlees and Engle (2017) where it is defined as the expected return of a financial firm's stock conditional on a market return being in its lower tail. According to this measure the firm's systemic risk contribution is function of tail dependence between market returns and a financial firm's stock returns, see e.g. Zhou and Tarashev (2013).
3. the Leverage ratio (*LVG*) is the ratio between the market value of equity and the book value of debt, over the market value of equity. This is the 'quasi-market value of leverage' as defined in Engle et al. (2015), and follows the definition on leverage as in Adrian and Shin (2010).
4. $\beta \times$ MV is the product of the time-varying beta estimate and the firm's market capitalization. The time-varying β coefficient is estimated following Engle (2015). This measure gives an estimate of the nominal (absolute) risk of the firm's market capitalization to systematic (market) shocks.
5. $\Delta CoVaR$ is defined as the *VaR* of the financial system, usually approximated by a market index, conditional on a certain institution being in distress. $\Delta CoVaR_i$ is defined as the *VaR* of the financial system when institution i is in distress, minus the *VaR* of the system when institution i is at its median value; details on this index can be found in Adrian and Brunnermeier (2016).

[1] We thank Federico Nucera for sharing with us the original dataset on which the paper Nucera et al. (2016) is based. We restrict our sample to include only EU institutions and we exclude real estate and other non financial corporation.

[2] http://vlab.stern.nyu.edu.

[3] http://www.esrb.europa.eu/pub/rd/html/index.en.html.

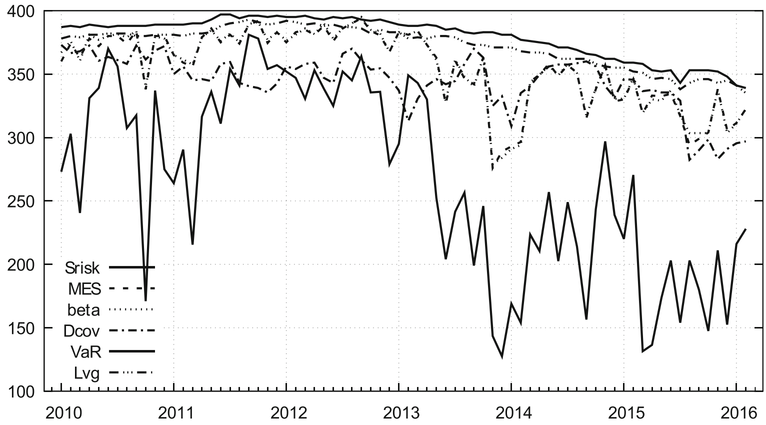


Fig. 3 Crédit Agricole: monthly risk measure indicators Jan. 2010–Dec. 2015

6. The value-at-risk (*VaR*) of an institution is an intermediate output of the calculation of *CoVaR*, see Acharya and Steffen (2015).

Table 3 in the Appendix collects the dataset summary statistics.

As an example in Fig. 3 we report the values of six indicators over the sample period for Crèdit Agricole where it is evident that the index *VaR* is more volatile than the other measures.

5 Results

5.1 Identification of risk classes

For each month the original data are collected in a $N \times 6$ matrix on which the PCA is performed. Specifically, we extract the first PC which explains between 42 and 60% of the total variance for each period, a result consistent with Nucera et al. (2016). Then the first PC on average explains at least more then 50% of the information on risk collected by the 6 indicators. In panel A of Fig. 4 it is reported the distribution of the percentage of total variance explained by the first eigenvalue over the sample with the superimposed Gaussian distribution (with sample mean and variance) and the usual Jarque–Bera test for normality.

The first principal components that account alone for not less than 40% of the overall variability are to be considered very meaningful. Panel B shows the variance explained by the fist eigenvalues over time. It is evident that the percentage of the explained total variance decreases around the first months of 2013 in correspondence of the Sovereign crisis in the euro area. While the first PC explain a significant percentage of the total variance, the overall decrease would support the inclusion of additional PC in the analysis. However there is a clear trade off between the advantage of using a single indicator for the classification and comprehensiveness resulting from more then one indicator.

The six systemic risk measures are positively correlated with each other and with the first principal component but they do not contribute in the same way to PC variance as illustrated by Table 1 and their contribution varies over time. In particular the table shows that $\beta \times$ MV

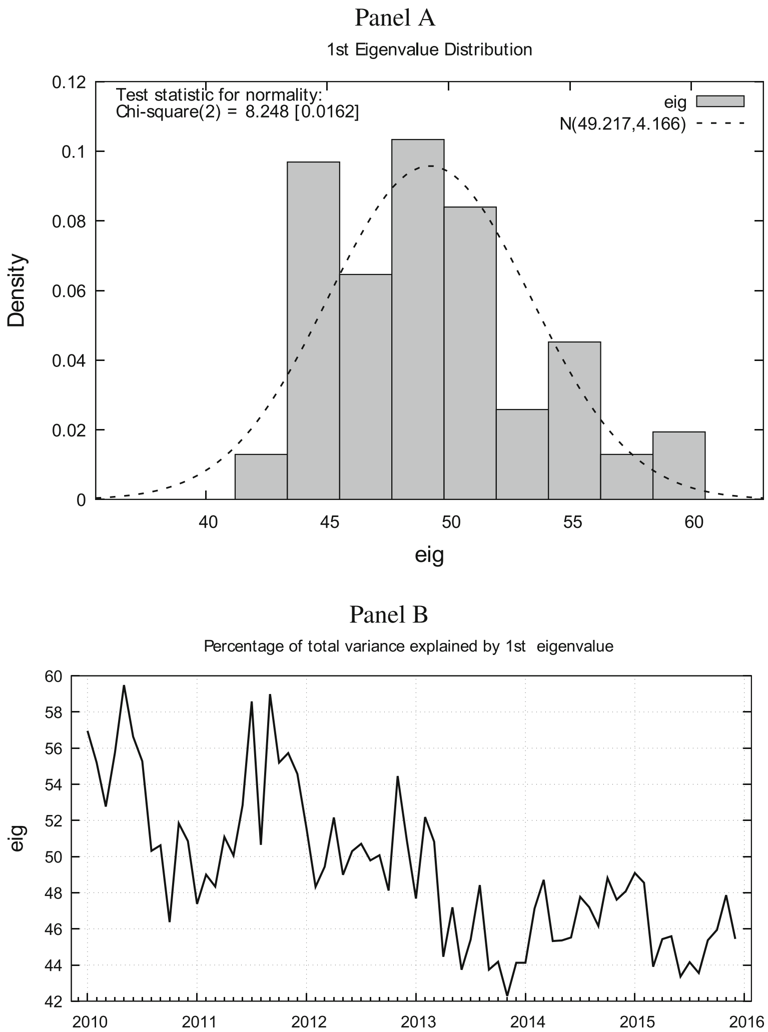


Fig. 4 Percentage of total variance explained by 1st PC

and *MES* (which are highly correlated) are—across months and years—the most important indicators, followed by ΔCoVaR. The *VaR* contribution falls starting from 2013.

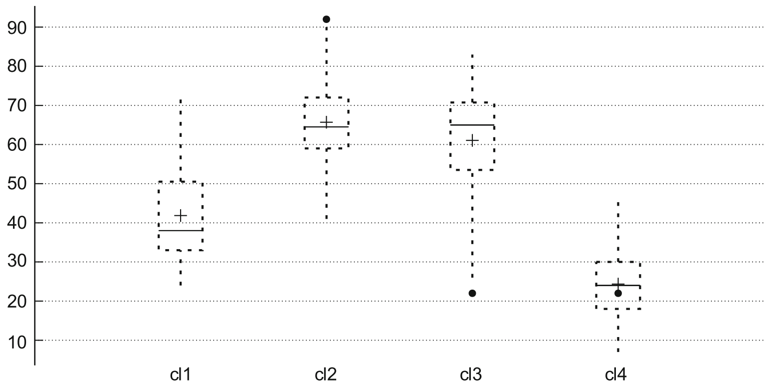
For each month we implemented the ART algorithm as described in Sect. 2. The ordered first principal component obtained on the $N \times 6$ data matrix of risk indexes plays the role of response variable Y which is, then, tree regressed on the artificial covariate k setting 5 as the minimum for the class size. We found that the optimal partition is into $G = 4$ groups. We labeled this groups as low (1), medium-low (2), medium-high (3) and high risk (4). Table 2 displays the summary statistics of the class dimension in total sample and for the sub-period 2010–2012 and 2013–2015. In Fig. 5 the main characteristics of the monthly group sizes are summarized by a box-plot. Observations outside the 95%—whiskers range are considered outliers and represented via dots. The high risk class shows a small size with an average dimension of 24 units. The 50% of the monthly class size belong to [18–30], this implies

Table 1 Correlation between risk indicators and 1st PC

Year	Beta	ΔCoVaR	Lvg	MES	SRisk	VaR
2010	0.95	0.66	0.53	0.95	0.58	0.53
2011	0.96	0.58	0.50	0.95	0.56	0.60
2012	0.96	0.46	0.45	0.96	0.56	0.63
2013	0.95	0.61	0.21	0.95	0.52	0.28
2014	0.96	0.73	0.14	0.96	0.51	0.32
2015	0.94	0.78	0.13	0.94	0.49	0.00

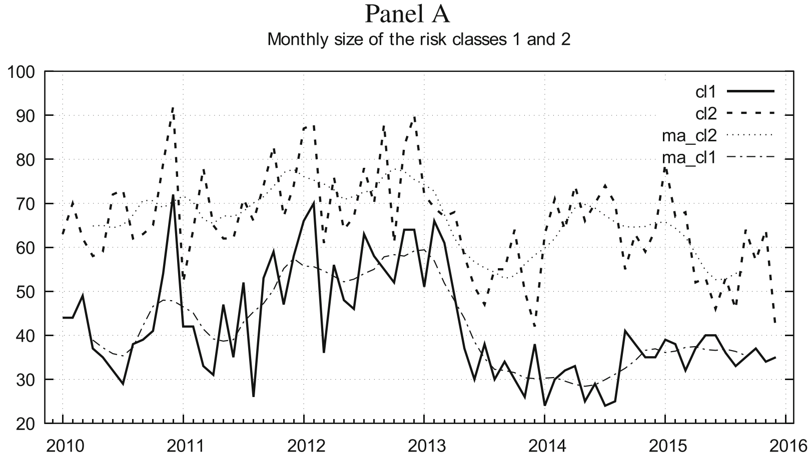
Table 2 Summary statistics of the risk classes

Class	Mean	SD	Min	Max
2010–2015				
cl1	41.86	12.19	24	72
cl2	65.69	11.19	41	92
cl3	61.07	14.7	22	83
cl4	24.35	9.57	7	53
2010–2012				
cl1	47.69	12.19	26	72
cl2	70.81	10.26	52	92
cl3	54.89	16.88	22	83
cl4	19.58	6.805	7	34
2013–2015				
cl1	36.03	9.085	24	66
cl2	60.58	9.746	41	79
cl3	67.25	8.66	43	83
cl4	29.11	9.633	12	53

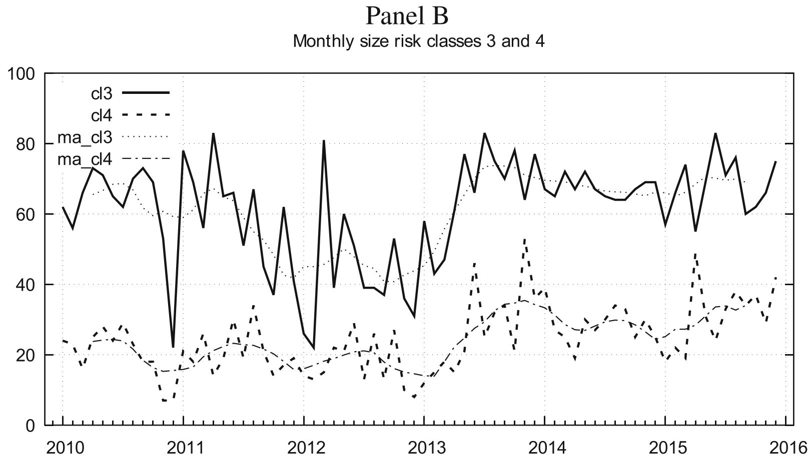


*cl*1: low risk; *cl*2: medium-low risk; *cl*3: medium-high risk; *cl*4: high risk ; + indicates the mean, outliers represented by dots.

Fig. 5 Box plots of monthly risk class sizes, Jan 2010-Dec 2015



*cl*1: low risk; *cl*2: medium-low risk; *ma_cl*1: and *ma_cl*2: centered 6 term moving average

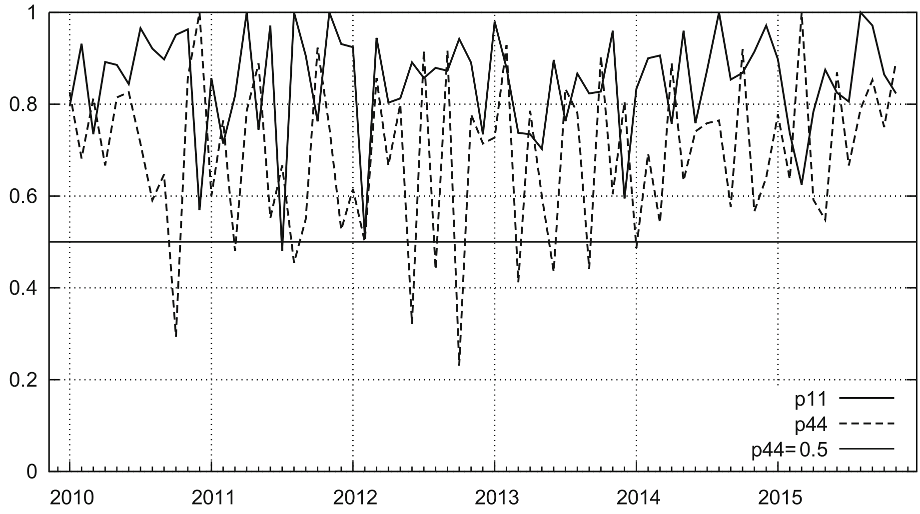


*cl*4: high risk; *cl*3: medium-high risk; *ma_cl*3: and *ma_cl*4: centered 6 term moving average

Fig. 6 Monthly risk class sizes, Jan 2010–Dec 2015

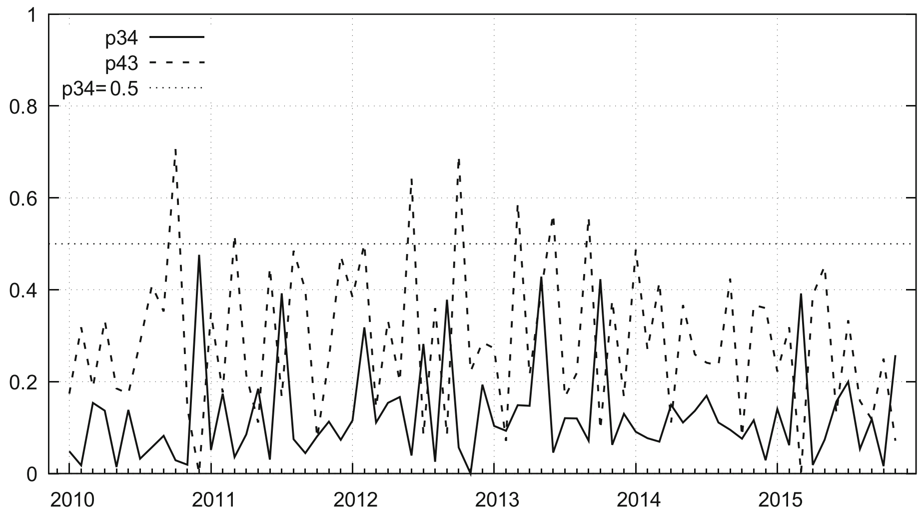
that the high risk class includes from 9 to 15% of the sample in 50% of time observations. On the other hand the size of the classes 2 and 3 is significant larger and more volatile. In the "Appendix" we provide the same information for each year of the sample, see Fig. 9. We can see that the number of firms belonging to the high risk class became much more volatile from 2013 onwards.

Figure 6 shows the size of the risk classes over time. In the figure we plot also a centered 6-term moving average of the size class series to underline their general behavior. Panel A shows that the size of the least risky classes drops significantly around the time of the Sovereign crisis. At the same time the size of class 3 and 4 increases (Panel B). It is notable that the size of the riskiest class 4 is larger than 20 on average and reaches more than 40 units after 2013 in few instances. This supports our claim that considering only the top 10 or top 15 of riskiest financial institution as a measure of the risk of the system maybe misleading.



p11=prob. to remain in the lowest risk class;
p44=prob. to remain in the highest risk class.

Fig. 7 Monthly transition probabilities, 2010–2015



p34=prob. to go to the higher risk class 4 starting from medium-high class 3;
p43=prob. to go to the medium-high risk class 3 starting from high risk class 4

Fig. 8 Monthly transition probabilities, 2010–2015

In the "Appendix" it is provided more detailed information on the behavior of the cut-off points in between the classes over the time period, see Fig. 10, and the mean and the standard error of the PC scores for each risk group and month, see Table 4.

When one or more indicators showed a peculiar behavior (extreme values) for some financial institutions in one or more months, they have been treated as supplementary individuals, that is they were not used for the determination of the principal components and their scores were predicted using the information provided by the performed PCA on the active individuals. Note that the number of these financial institutions is small (from 1 in 2011 to 9 in 2015) and only in few cases they were treated as supplementary individuals for most part of the year as detailed in Table 5 of the "Appendix". However, these events, involving a limited number of institutions, are to be considered as due to the specific financial situation of those institutions rather than to a general moment of distress in the market as a whole.

5.2 Transition probability

A natural outcome of our analysis is to calculate the probability that a financial institution moves from a risk class to another one. Following Jones (2005), we estimate the Markov Transition Matrices as follows. Let n_{ij} be the number of units that were in class i in period $t-1$ and are in class j in period t. The estimated probability of financial institution being in state j in period t conditional on it been in state i in period $t-1$ is given by $p_{ij} = \frac{n_{ij}}{\sum_j n_{ij}}$, i.e. the units that started in state i and ended in state j as a proportion of sum of units in state i at time t. In Fig. 7, we report the estimated monthly probabilities p_{11} and p_{44}, which are the probabilities to remain in the lowest and in the highest risk class. In Fig. 8 we report the estimated monthly probabilities p_{34} and p_{43}, which are the probabilities to go to the higher risk class starting from class 3, or from class 4 to go down into class 3. Even in these cases we plot also a centered 6-term moving average of the series to underline their general behavior. From the first figure we see that p_{11} and p_{44} are in general over 0.5. It should keep in mind that the relative high volatility of the p_{44} results from the small size of this class.

This analysis could help in choosing the institution to be monitored. Since the probability to remain in the lowest risk class is always more than 50%, supervisor may decide to monitoring these institutions less frequently. On the other hand financial institution in the highest risk class should be monitored on continuous basis. In our sample just one financial institution remains in class 4 for the entire period and 9 firms remain in that class more then 75% of the time periods. In January 2010, 24 firms are in class 4: in particular 19 are banks; in December 2015, 42 firms, including 23 banks, are in class 4.

6 Conclusions

We have proposed a novel approach to classify financial institutions (banks) into homogeneous groups based on a synthetic indicator of risk. This approach provides a flexible way to identify groups of risky financial institutions. At the same time, it could give policy makers an easy and intuitive instrument to evaluate periods in which the level of systemic risk is high, depending on the size of the riskiest group(s). Note that, opposed to our procedure, standard classification tools such as cluster analysis, do not provide neither a synthetic risk indicator nor an ordering of the observations and moreover the identified partition into groups could not be used for prediction purposes i.e. to assign a financial firm to a risk group.

In our application, we also show that riskier financial institutions tend to remain in the same group with very high probability also over the successive periods, which may provide very useful indications to supervisors in charge of micro-prudential supervision.

Appendix

Table 3 collects the descriptive statistics of the risk indicators. Table 4 refers to the mean and the standard error of the principal component scores for each risk group and month. Figure 9 describes the characteristics of the class size distribution for each year, while the behavior of the cut-off points over the time period are reported in Fig. 10 (Table 5).

Table 3 Systemic risk indicators descriptive statistics

Stat	Beta	Dcov	LVG	MES	SRisk	VaR
2010						
Min	0.12	0.01	1.02	0.31	– 14,992.00	10.20
Max	3.59	0.71	403.66	8.79	130,245.00	589.30
Mean	1.03	0.43	17.05	2.83	6532.82	37.76
SD	0.43	0.13	27.10	1.18	19,857.82	25.51
2011						
Min	– 0.02	– 0.01	1.02	– 0.08	– 17,323.00	10.00
Max	3.40	0.73	9280.12	11.87	154,897.00	342.80
Mean	1.10	0.43	31.39	3.18	7263.95	43.11
SD	0.49	0.13	290.17	1.44	21,475.12	25.95
2012						
Min	0.10	0.04	1.01	0.25	– 17,151.00	9.80
Max	6.70	0.74	2974.42	17.89	152,556.00	614.60
Mean	1.36	0.44	29.06	3.55	7990.68	39.42
SD	0.69	0.13	109.88	1.77	22,741.55	26.24
2013						
Min	0.02	– 0.04	1.01	0.05	– 34,284.00	8.30
Max	5.58	0.71	10,561.62	14.32	129,931.00	435.90
Mean	1.16	0.38	46.88	2.93	5808.82	34.39
SD	0.52	0.14	449.65	1.31	19,298.35	29.73
2014						
Min	– 0.10	– 0.23	1.01	– 0.27	– 51,894.00	6.60
Max	3.11	0.74	5433.48	7.67	109,848.00	250.40
Mean	1.03	0.33	31.41	2.60	4122.18	29.55
SD	0.44	0.14	278.52	1.10	16,584.94	16.60
2015						
Min	– 1.38	– 0.27	– 0.34	– 3.51	– 136,696.00	9.30
Max	6.34	0.75	5141.23	16.04	109,849.00	519.00
Mean	0.96	0.29	29.13	2.52	3709.24	44.42
SD	0.57	0.19	222.75	1.45	15,834.01	41.59

Table 4 Mean value in recursive partition (*standard error in italic*)

cl.	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2010												
1	− 0.782	− 0.804	− 0.834	− 0.764	− 0.675	− 0.704	− 0.824	− 0.688	− 0.658	− 0.641	− 0.795	− 0.785
	0.111	*0.119*	*0.081*	*0.124*	*0.126*	*0.123*	*0.103*	*0.152*	*0.148*	*0.137*	*0.085*	*0.095*
2	− 0.457	− 0.409	− 0.599	− 0.387	− 0.263	− 0.289	− 0.396	− 0.257	− 0.284	− 0.258	− 0.550	− 0.500
	0.097	*0.103*	*0.058*	*0.103*	*0.104*	*0.111*	*0.111*	*0.106*	*0.101*	*0.107*	*0.076*	*0.078*
3	− 0.081	− 0.034	− 0.377	− 0.019	0.105	0.106	− 0.021	0.155	0.073	0.150	− 0.272	− 0.175
	0.118	*0.136*	*0.082*	*0.125*	*0.119*	*0.102*	*0.098*	*0.119*	*0.134*	*0.137*	*0.118*	*0.117*
4	0.447	0.523	0.034	0.509	0.625	0.606	0.500	0.654	0.632	0.626	0.394	0.384
	0.220	*0.228*	*0.276*	*0.202*	*0.190*	*0.198*	*0.240*	*0.189*	*0.207*	*0.171*	*0.322*	*0.376*
2011												
1	− 0.736	− 0.718	− 0.625	− 0.821	− 0.618	− 0.6896	− 0.638	− 0.616	− 0.618	− 0.693	− 0.778	− 0.649
	0.100	*0.104*	*0.142*	*0.060*	*0.138*	*0.117*	*0.120*	*0.150*	*0.141*	*0.110*	*0.096*	*0.138*
2	− 0.438	− 0.462	− 0.263	− 0.666	− 0.265	− 0.372	− 0.304	− 0.208	− 0.254	− 0.354	− 0.477	− 0.247
	0.080	*0.065*	*0.091*	*0.047*	*0.083*	*0.086*	*0.101*	*0.105*	*0.106*	*0.093*	*0.082*	*0.102*
3	− 0.136	− 0.212	0.030	− 0.499	0.050	− 0.027	0.079	0.155	0.098	0.004	0.004	− 0.211
	0.108	*0.087*	*0.081*	*0.059*	*0.123*	*0.108*	*0.121*	*0.104*	*0.122*	*0.104*	*0.103*	*0.145*
4	0.347	0.181	0.360	− 0.246	0.515	0.377	0.561	0.561	0.628	0.553	0.326	0.711
	0.146	*0.240*	*0.103*	*0.061*	*0.192*	*0.186*	*0.171*	*0.166*	*0.224*	*0.129*	*0.222*	*0.192*
2012												
1	− 0.775	− 0.781	− 0.614	− 0.787	− 0.699	− 0.710	− 0.714	− 0.757	− 0.746	− 0.708	− 0.804	− 0.785
	0.103	*0.085*	*0.126*	*0.109*	*0.145*	*0.121*	*0.128*	*0.126*	*0.122*	*0.141*	*0.091*	*0.105*
2	− 0.442	− 0.552	− 0.279	− 0.466	− 0.311	− 0.338	− 0.351	− 0.364	− 0.366	− 0.301	− 0.540	− 0.487
	0.098	*0.087*	*0.109*	*0.090*	*0.117*	*0.109*	*0.096*	*0.110*	*0.110*	*0.099*	*0.074*	*0.089*

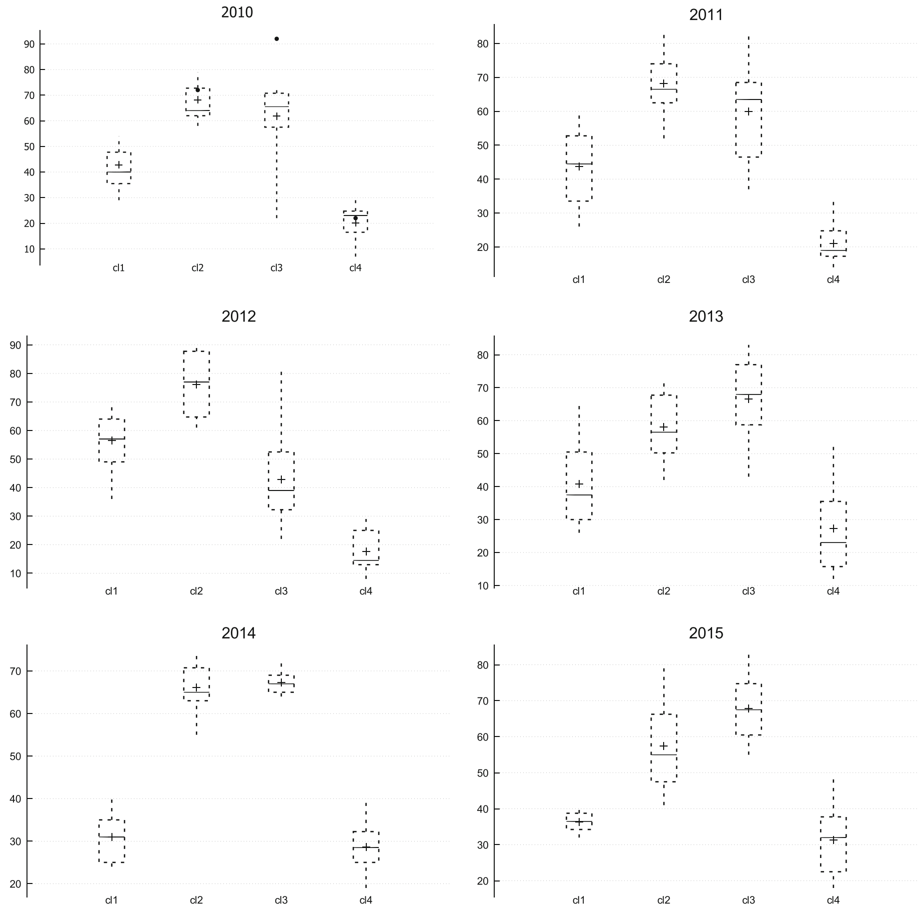
Table 4 continued

cl.	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
3	− 0.038	− 0.109	0.147	− 0.136	0.130	0.024	0.070	0.043	0.086	0.047	− 0.228	− 0.141
	0.138	*0.101*	*0.149*	*0.080*	*0.140*	*0.109*	*0.139*	*0.126*	*0.125*	*0.126*	*0.103*	*0.132*
4	0.517	0.389	0.735	0.359	0.632	0.521	0.625	0.528	0.522	0.522	0.408	0.132
	0.234	*0.242*	*0.186*	*0.213*	*0.182*	*0.198*	*0.211*	*0.245*	*0.190*	*0.201*	*0.293*	*0.344*
2013												
1	− 0.752	− 0.731	− 0.764	− 0.703	− 0.604	− 0.661	− 0.611	− 0.618	− 0.638	− 0.621	− 0.425	− 0.657
	0.120	*0.118*	*0.105*	*0.109*	*0.125*	*0.144*	*0.138*	*0.122*	*0.129*	*0.147*	*0.165*	*0.139*
2	− 0.396	− 0.407	− 0.453	− 0.390	− 0.256	− 0.229	− 0.278	− 0.255	− 0.240	− 0.166	− 0.030	− 0.236
	0.097	*0.091*	*0.088*	*0.075*	*0.097*	*0.105*	*0.088*	*0.105*	*0.095*	*0.110*	*0.103*	*0.094*
3	− 0.057	− 0.049	− 0.118	− 0.097	0.112	0.141	0.084	0.094	0.119	0.231	0.309	0.132
	0.141	*0.136*	*0.129*	*0.116*	*0.125*	*0.095*	*0.130*	*0.106*	*0.112*	*0.129*	*0.087*	*0.121*
4	0.613	0.480	0.439	0.441	0.634	0.553	0.557	0.514	0.491	0.713	0.649	0.579
	0.242	*0.206*	*0.209*	*0.233*	*0.204*	*0.167*	*0.200*	*0.181*	*0.173*	*0.165*	*0.131*	*0.167*
2014												
1	− 0.642	− 0.641	− 0.706	− 0.597	− 0.669	− 0.605	− 0.655	− 0.674	− 0.693	− 0.626	− 0.609	− 0.518
	0.138	*0.131*	*0.122*	*0.144*	*0.107*	*0.097*	*0.139*	*0.125*	*0.126*	*0.125*	*0.123*	*0.144*
2	− 0.222	− 0.284	− 0.362	− 0.151	− 0.198	− 0.160	− 0.205	− 0.206	− 0.356	− 0.254	− 0.224	− 0.104
	0.114	*0.110*	*0.098*	*0.112*	*0.126*	*0.099*	*0.113*	*0.123*	*0.086*	*0.097*	*0.094*	*0.098*
3	0.140	− 0.054	− 0.007	0.239	0.192	0.216	0.189	0.192	− 0.042	0.118	0.118	0.283
	0.098	*0.098*	*0.117*	*0.118*	*0.114*	*0.102*	*0.118*	*0.115*	*0.090*	*0.113*	*0.095*	*0.119*
4	0.544	0.455	0.478	0.713	0.647	0.623	0.610	0.597	0.325	0.574	0.558	0.737
	0.163	*0.187*	*0.212*	*0.131*	*0.135*	*0.157*	*0.173*	*0.154*	*0.119*	*0.147*	*0.180*	*0.150*

Table 4 continued

cl.	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2015												
1	−0.676	−0.594	−0.720	−0.590	−0.475	−0.644	−0.711	−0.644	−0.646	−0.596	−0.702	−0.706
	0.140	*0.147*	*0.107*	*0.160*	*0.160*	*0.139*	*0.139*	*0.145*	*0.1434*	*0.158*	*0.121*	*0.121*
2	−0.245	−0.194	−0.371	−0.208	−0.138	−0.316	−0.330	−0.281	−0.204	−0.143	−0.338	−0.336
	0.115	*0.098*	*0.093*	*0.114*	*0.083*	*0.094*	*0.116*	*0.088*	*0.115*	*0.114*	*0.103*	*0.103*
3	0.178	0.175	0.012	0.148	0.207	−0.066	−0.056	0.068	0.169	0.223	0.018	0.005
	0.130	*0.120*	*0.122*	*0.092*	*0.095*	*0.116*	*0.116*	*0.101*	*0.122*	*0.124*	*0.116*	*0.108*
4	0.762	0.698	0.582	0.494	0.562	0.536	0.507	0.466	0.593	0.645	0.456	0.416
	0.160	*0.214*	*0.236*	*0.156*	*0.166*	*0.209*	*0.199*	*0.167*	*0.172*	*0.155*	*0.223*	*0.174*

*cl*1: Low risk; *cl*2: medium-low risk; *cl*3: medium-high risk; *cl*4: high risk



$cl1$: low risk; $cl2$: medium-low risk; $cl3$: medium-high risk; $cl4$: high risk

Fig. 9 Box plots of monthly risk class sizes by year

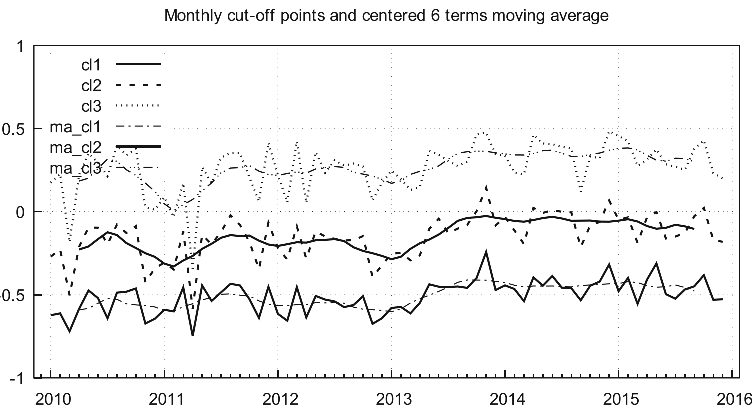


Fig. 10 Monthly cut-off points, 2010–2015

Table 5 Outliers by year in brackets the number of months out of range

2011	2012	2013	2014	2015
Permanent TSB (3)	Dexia (3)	Allied Irish Banks (1)	Banca Popolare dell'Emilia Romagna (2)	van Lauschot (1)
	Unipol (2)	Attica (1)	Cassa di Risparmio di Genova (2)	Banco Sabadell (1)
		National Bank of Greece (1)	Dexia (2)	Oderbank (1)
		Marfin Holding (1)	MPS (2)	Banca Transilvania (1)
		Piraeus Bank (1)		Piraeus Bank (10)
		Eurobank Ergasias (3)		OTP Bank (1)
				Alpha bank (2)
				Eurobank ergasias (8)
				National Bank of Greece (7)

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