

# Quadratic cavity soliton optical frequency combs

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**Abstract:** We theoretically investigate the formation of frequency combs in a dispersive second-harmonic generation cavity system, and predict the existence of quadratic cavity solitons in the absence of a temporal walk-off. © 2019 The Author(s)

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There is currently much interest in the generation of broadband optical frequency combs (OFCs) by means of nonlinear third-order or Kerr cavities [1]. It has been recently demonstrated that OFCs may also be generated in quadratic cavities, which permits comb generation in spectral regions where no suitable pump sources are available, or where the material dispersion properties prevent Kerr comb generation [2, 3]. One of the main features of Kerr OFCs is the possibility of achieving coherent cavity soliton (CS) based combs, which correspond to broadband, coherent and mode-locked temporal pulses with a fixed repetition rate [4].

For quadratic combs to become a viable alternative to Kerr combs, it would be important to demonstrate if they may also exhibit CS solutions. Our work reveals, we believe for the first time, the existence of stable mode-locked CS in a dispersive second-harmonic generation (SHG) based cavity system.

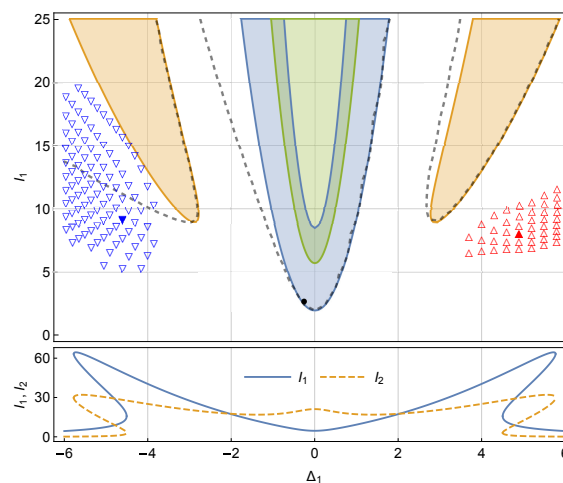


Fig. 1. Main panel: Stability phase diagram for dimensionless loss coefficient  $\alpha = 1$  and SH and FF cavity detuning  $\Delta_2 = 2\Delta_1$  [5]. The homogeneous solutions are unstable to cw perturbations within the shaded regions and modulationally unstable within the dashed contour. Bottom panel: Variation in FF/SH intracavity power for the homogeneous solution with driving strength  $S = 12$ .

Specifically, we consider the formation of quadratic combs in a dispersive SHG cavity with resonant fundamental field (FF) at frequency  $\omega_0$  and second harmonic (SH) field at  $2\omega_0$  [5]. Comb generation in this system relies on the initial frequency doubling of the driven FF to create a second-harmonic wave. The SH is in turn the source of an internally pumped optical parametric oscillator (OPO) that results in the growth of subharmonic sidebands above a

certain pump threshold. Subsequent, cascaded three-wave mixing interactions among the different components results in the formation of simultaneous combs around both fundamental and SH wavelengths. Moreover, we consider a cavity enclosing a quasi-phase matched LiNbO<sub>3</sub> crystal, with the FF and SH wavelengths on opposite sides of the zero-dispersion wavelength, so that their temporal walk-off is canceled, and SHG phase-matching is achieved by periodic poling.

Figure 1 shows the stability phase diagram of cavity steady-state solutions in the parameter space ( $\Delta_1, I_1$ ), where  $\Delta_1$  and  $I_1$  are the FF cavity detuning and intensity, respectively. The different instability regions for perturbations with frequency  $\Omega = 0$  are colored. Each point on the diagram corresponds to a comb state that can be realized by the stationary homogeneous solution for a particular combination of pump power and detuning. Orange shaded regions mark domains that correspond to the unstable middle branch of the bistable homogeneous solution, while the system has complex conjugated eigenvalues and may display self-pulsing within the blue shaded region. The blue and green regions are seen to partially overlap, with the eigenvalues ceasing to be oscillatory at the upper boundary of the blue domain and becoming purely real within the non-overlapping green region. The boundaries of the domains that exhibit modulational instability (MI) to periodic perturbations are additionally marked in the figure with a dashed contour.

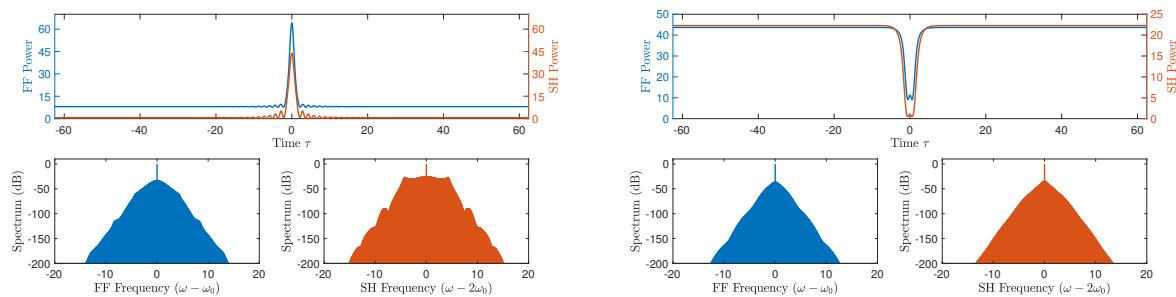


Fig. 2. Left panel: Normalized intracavity power (top) and spectra (bottom) of a bright cavity soliton for driving strength  $S = 12$  and detuning  $\Delta_1 = 4.9$  (marked by a filled red upwards-facing triangle in Fig. 1). Right panel: Example of a stable localized dark soliton solution for driving strength  $S = 10$  and detuning  $\Delta_1 = -4.6$  (marked by a filled blue downwards-facing triangle in Fig. 1).

Using the phase diagram, we predict the location of CSs in the parameter space. An example of a bright CS solution is shown in the left panel of Fig. 2. Both the FF and SH amplitudes are seen to display a localized central peak with damped oscillations on either side. We emphasize that the signs for group velocity dispersion of the two fields differ. Whereas an example of a dark localized structure is shown in the right panel of Fig. 2. Dark CSs correspond to holes in the modulationally stable upper branch homogeneous solution, where the intracavity power is locally reduced.

In conclusion, we have shown that quadratic OFCs in a doubly-resonant SHG cavities may lead to bright and dark localized CS solutions. These combs have analogous properties to Kerr CSs, which suggests that quadratic frequency combs may be a viable alternative to Kerr combs, with unique benefits for a variety of applications.

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