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The differentially-tilted toroidal field coil concept for tokamaks

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ABSTRACT

The implications of the adoption of a tokamak's toroidal field coil characterized by differential tilting in the azimuthal direction are investigated. From an engineering point of view, the major advantage introduced by such coils is a drastic reduction of some components of the electromagnetic forces in certain areas. As a beneficial side-effect, they generate poloidal field, in addition to toroidal field. The former advantage allows for a partial relaxation of the reinforcing structural material required in the machine design, while the poloidal flux generated during the current rise, when used in conjunction with the conventional central solenoid, would allow for discharges of longer duration. This paper presents results obtained by applying the tilting optimization procedure to circular and D-shaped coils, and characterized by geometrical and physical parameters proper of high-field compact tokamaks, since the issue of electromagnetic force reduction is most relevant in these devices.

1. Introduction

Tokamaks have proven to be the most successful configuration for the magnetic confinement of plasmas. The design of viable fusion reactors, however, remains largely elusive at this time, since both physics and engineering issues limit device performance: large volume, low/intermediate field tokamaks (i.e., where the ratio of the toroidal field on axis in T and the major radius in m is $B_T/R_0 \lesssim 1$) cannot reach the high fusion gain that reactors require, while compact, high field devices with conventional non-superconductive copper coils, although more suitable to reach ignition regimes, have too low a duty cycle to be commercially attractive.

The tilted coil concept discussed here provides a way to combine the advantages of high toroidal field and improved duty cycle. The concept originates from an idea presented over thirty years ago, first in the form of a patent [1], and then of two papers [2,3] dealing with the general issues associated to the generation of large toroidal magnetic fields in compact tokamak devices, and describing the characteristics and mechanical properties of the so-called "tilted-winding" coil. A similar concept was described in [4,5]. Fig. 1 reproduces the original figure of the tilted coil presented in Ref. [2].

The original idea is to give the toroidal field coil (TFC) a tilt in the horizontal direction on the inboard side. The coil section shown in Fig. 1 starts on the outboard mid-plane, it wraps over the top of the torus cavity, then it is given an approximately 45° tilt as it follows down the inboard section, to eventually re-emerge on the outboard side.

The immediate advantage of this solution is that stresses in the inner leg of the toroidal field (TF) magnet are relieved. Mechanically unloading the TFC makes it easier to generate the fields required to approach fusion burning conditions at higher density and relatively lower temperatures. In particular, the heavy steel structure needed to withstand the huge electromagnetic (EM) forces in conventional magnets could be considerably reduced. In perspective, this type of coil could be made of ribbons of high-temperature superconductors, characterized by high critical magnetic fields but rather poor structural properties. However, technological advances are making them attractive for planned machines so far designed with TFCs of conventional geometry, like for example the SPARC tokamak project put forth by Commonwealth Fusion Systems (CFS) in collaboration with the MIT Plasma Science and Fusion Center [6].

As a side-effect, the new coil generates poloidal field, in addition to toroidal field. It could therefore provide some of the flux swing for generating the plasma (which represents most of the V-s consumption), thus allowing the discharge to be sustained for a longer time.

In this work we use numerical computation to compare the EM forces acting on the various regions of a conventional toroidal field coil with the forces acting on a coil that has some or all of its sections tilted. The effect of tilting depends on the location of the section under consideration: for example, tilting the inner and respectively the outer section of the coil leads to very different modifications of the EM forces acting on them. Furthermore, the shape of the coil and its relative size (i.e., the torus aspect ratio) leads to different results.

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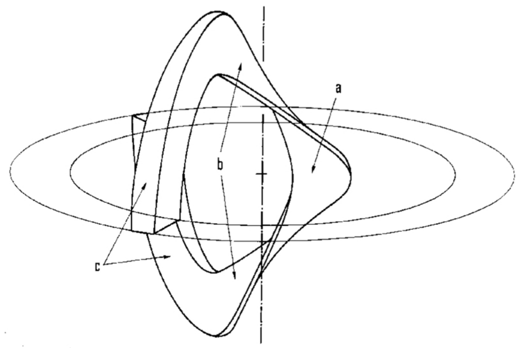


Fig. 1. The toroidal field "tilted-winding" coil presented in [2]. Legend: a: inner region; b: transition region; c: outer region.

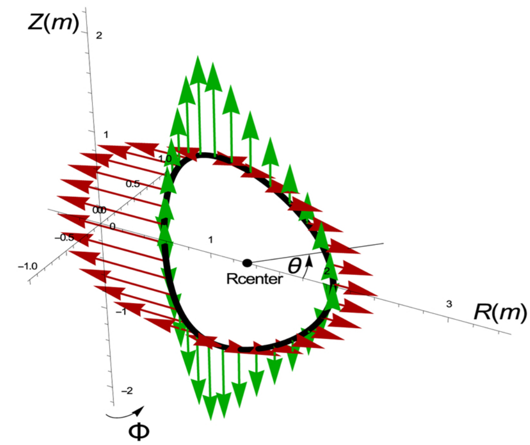


Fig. 2. Radial and vertical forces acting on a conventional D-shaped toroidal field coil, due to the interaction of the coil current with the self-induced magnetic field (arbitrary units). The center of the coils, R_{center}, coincides with the major radius R_0.

Starting from 1994 up to 2011 a series of papers dealing with the concept of the quasi-force-free helical winding have been published by Japanese authors ([7–11], to cite a few). While the guiding objective is similar to ours, we follow a different methodology, that is, we look for configurations that optimally tilt the region of the coil under consideration, thus obtaining a tilting that varies around the coil (hence the name "differentially-tilted" coil). Very interestingly, the authors of these papers actually built and tested small tokamak models, thus establishing a proof-of-principle for these new coil concepts, including plasma formation. The adoption of various types of helical coils were proposed also in the context of spherical tokamaks, such as the sphellamak device [12,13].

Devices obtained giving a uniform tilt to the coil so that it preserves its planarity have also been proposed and studied [14–18]. Numerical simulation and preliminary experimental results indicate that these "tokamak-torsatron" hybrid devices can generate rotational transform with less plasma current than conventional tokamaks, if not zero current at all, with interesting implications for disruptions and steady-state operation. The line of research based on planar coils, which includes applications to energy storage systems [19], is however fundamentally different from the one pursued in this paper.

The remaining of the paper is organized as follows. In the next section we present a brief overview of the EM forces acting on the toroidal field coils of a tokamak device, the problems associated with them, and outline the effect of differential coil tilting on the forces. This section also provides a description of the numerical code specifically written to perform the tilting optimization studies. In Section 3 we consider a toroidal field coil of circular shape. The EM forces acting on it are minimized, and the simplest possible model for the plasma current, i.e., a current loop centered on the torus axis, is introduced. Moreover, we add the poloidal field coils, again modeling them with current loops centered on axis, and compare the magnetic field streamlines of the conventional and differentially-tilted configuration. The D-shaped TFC model is considered in Section 4, where emphasis is posed on the optimization of only the inner leg. It is important to stress that the magnetic field surfaces inside the plasma region obtained by the simple plasma current loop are obviously not realistic. The purpose to show them is for comparison between conventional and differentially-tilted configurations, to give a glimpse of potential problematic consequences of tilting on various issues of plasma equilibrium and stability. Possible operational scenarios adopted for differentially-tilted coil tokamaks are suggested in Section 5. A summary and conclusion section ends the paper.

2. The numerical code

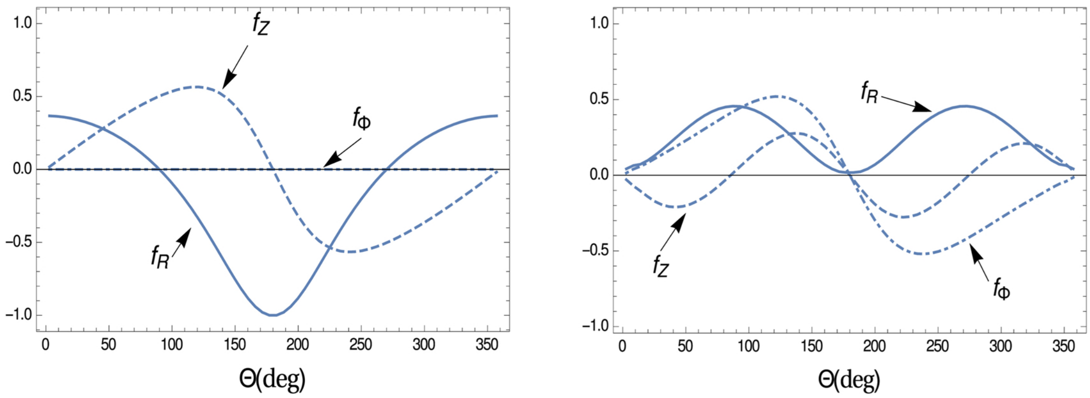
The magnetic confinement of high-temperature plasmas in a tokamak device is realized mainly with three sets of coils, (i) the toroidally symmetric TFC system which encloses the plasma chamber, a "doughnut" shaped metal vessel, (ii) the central solenoid (CS) housed in the bore of the doughnut, and (iii) the external poloidal field coil (PFC) system. The total current I flowing in the TFC generates a toroidal field $B_\Phi(\mathrm{T}) = I(\mathrm{MA})/(5R(\mathrm{m}))$ in the magnet cavity where the plasma vessel is located. The current swing of the CS induces a current I_p in the plasma loop, which acts as the secondary single coil of a transformer. This plasma current generates the poloidal field B_p, generally weaker than the toroidal field. The resulting field lines in the plasma region are helically wound around the torus. This magnetic configuration is however not sufficient to ensure plasma control: external PFCs are needed to provide the additional vertical field required for the stabilization, positioning and shape control of the plasma column.

The interaction of the current flowing in the coils with the magnetic field they generate gives rise to EM forces of tens of MN. In particular, the conducting plates of the TFCs experience centrifugal meridian forces that are highest on the inboard legs, where the toroidal field is strongest, as well as out-of-plane forces arising from the interaction of the magnet current with the poloidal field. In Fig. 2, the general features of the radial and vertical forces acting on a conventional D-shaped tokamak coil are illustrated.

Since the problem of the EM forces on the TFCs is exasperated in compact tokamaks aiming at producing large toroidal fields, we have adopted for our studies design parameters which are representative of these typology of machines. Various methods have been envisaged to contrast the EM forces, with the result of both a complex and expensive design and a limitation on the maximum toroidal field that can be produced. For example, the compact, high field device Ignitor [20] has to implement several solutions to cope with the EM forces in the toroidal magnet: "bucking and wedging" between the TF and CS magnet coils, a pre-loaded structural steel shell (C-clamps) for mechanical compression, in addition to an electromagnetic press.

In the present work the issue of force reduction on the TFCs is addressed numerically using a Mathematica [21] code specifically written to find the local tilting angle of any region of a TFC which minimizes the total EM force (or a single component) acting on it. The TFC can have any form, and in particular can be D-shaped as in present tokamak designs. The code is the upgraded version of a more rudimentary code that has already been used to perform a preliminary study on the effect of tilting of a TFC composed of four distinct legs, which in the non-tilted case would form a rectangle in the meridian plane [22].

To evaluate the magnetic field produced by a toroidal coil, the code discretizes it into many short straight wires, each one producing a magnetic field according to the formula: $\mathbf{B} = (\mu_0 I/4\pi d)(\cos\theta_1 + \cos\theta_2)\hat{\boldsymbol{n}}$, where I is the current flowing in the wire, d is the



(a) *Radial and vertical forces in the non-tilted circular coil. The toroidal component of the force is identically zero.*

(b) *Radial, toroidal and vertical forces in the circular coil differentially-tilted for minimum total force.*

Fig. 3. Comparison of radial, vertical and toroidal forces per unit wire length. The values have been normalized to the maximum value obtained in the non-tilted case ($f_{\max} = f_{R,max} = 5.147\ 10^7$ N/m). The angle Θ identifies the center-point of the wires, anticlockwise from the outboard mid-plane (see Fig. 2).

distance between the wire and the observation point (where the magnetic field is calculated), θ_1 and θ_2 are the angles between the wire and the two lines connecting the two end-points of the wire to the observation point, and finally $\hat{\boldsymbol{n}}$ is the unit vector perpendicular to the plane containing the wire and the observation point, and pointing in the direction of the magnetic field. Note that the field provided by this formula is singular on the wire itself. To evaluate the field at the location of a selected wire, required to find the EM forces acting on it, we therefore superimpose the magnetic field produced by all wires except the one under consideration. The approximation introduced by this procedure is small, since the wires are short due to their large number (e.g., a single coil is discretized in 64 or 128 straight wires).

To speed up numerical computation, instead of working with a continuous coil while differentially tilting its various regions, we keep fixed the center-point of each wire, and tilt the wire around it. This approach leads to optimized coils made of non-connected tilted wires [see, for example, the inner region of the coils in Fig. 7a]. The approximation introduced by this methodology does not invalidate the conclusions on force reduction presented in the present work, as is shown at the end of Section 4.

In the optimization studies, when we tilt a wire, we increase its length so to maintain the same Z coordinate of its end-points. In this way, the tilted wire produces poloidal field, while maintaining the same toroidal field as when the wire is non-tilted. It is therefore more reasonable to evaluate the force acting on the unit length of each wire. Indicating with $\hat{\mathbf{L}}$ the versor associated to the wire of length L, and with I the current flowing in it, we write $I\mathbf{L} = \mathrm{L}\,\mathrm{I}\hat{\mathbf{L}} = L\mathbf{I}$, where $\mathbf{I} \equiv I\hat{\mathbf{L}}$ is now a vector current. Then $\mathbf{F} = L\mathbf{I} \times \mathbf{B}$, and the force per unit length is given by

$$\mathbf{f} \equiv \mathbf{F}/L = \hat{\boldsymbol{R}}(I_\Phi B_Z \underline{-I_Z B_\Phi}) + \hat{\boldsymbol{\Phi}}(I_Z B_R - I_R B_Z) + \hat{\boldsymbol{Z}}(\underline{I_R B_\Phi} - I_\Phi B_R)\ , \qquad (1)$$

where the underlined terms are the only forces acting on a conventional (untilted) coil in isolation. The force evaluated in the code is based on this expression. The optimal tilting angle in the azimuthal direction, Γ, is chosen so to minimize one or more components of the force acting on a wire, or its modulus. The optimization procedure is iterative, in the sense that all wires in the upper plane ($Z > 0$; all our studies consider configurations with up-down symmetry) are optimized in sequence, until convergence.

The TFC is discretized in an even number of wires, usually 64 or 128, according to convergence requirement (if wires are long, a small tilting introduces large variations of the produced magnetic field, and convergence toward an optimized configuration is more difficult). We note that in the untilted configuration, the current in the TFCs flows in the anticlockwise ($+\theta$) direction, producing a toroidal field in the negative azimuthal ($-\Phi$) direction (for a definition of these angles, see Fig. 2). In the following optimization studies, the tilting of the coil wires will occur in the clockwise direction around the center point of the wire. As a consequence, each single rotated wire with center-point located at, say, R_{wire}, produces a vertical field that is positive (in the $+Z$ direction) in the region $R < R_{\mathrm{wire}}$, and negative in the region $R > R_{\mathrm{wire}}$.

To better clarify the computation procedure, we observe that when a specific wire is tilted, the wire symmetric with respect to the $Z = 0$ plane is tilted by the same amount, and this is done for all the coils in the system. This means that also the magnetic field at the center of the specific wire, used to find the optimal tilting angle, depends on Γ. The tilting angle which minimizes the force is found using constrained nonlinear optimization numerical algorithms provided by the Mathematica software.

3. The optimally-tilted coils with circular shape

3.1. TFC system in isolation

We begin considering a TFC system in isolation (i.e., not coupled with a plasma current and/or a PFC system) made of 24 coils of circular shape, centered at $R_{\mathrm{center}} = 1.32$ m and with a radius of $a_{\mathrm{coil}} = 0.74$ m. Circular coils were typical of the early tokamak experiments, such as the Alcator devices at MIT. On this configuration we explore the effect of tilting the wires all around the perimeter of the coil, thus extending the idea presented in Refs. [1–4].

We find that a circular coil whose wires are differentially-tilted in such a way as to minimize the total force per unit length, experiences very low forces, if any, on the central part of the inner and outer regions. The forces as a function of the poloidal angle before and after the optimization are shown respectively in part (a) and (b) of Fig. 3.

It can be observed how the radial force is very much decreased in the central part of the inner region of the coil (about one order of magnitude), and is now positive (outward directed) all around the coil. The vertical component is also significantly reduced in the upper (and lower) region of the coil, and changes its sign with respect to the non-tilted case in the outer region. As already pointed out, the price to pay for this force reduction is the insurgence of out-of-plane toroidal forces. The maximum magnitude of this new force component is slightly larger than the maximum magnitude of the remaining components, but never rises above half of the maximum value of the forces in the untilted configuration.

The presence of these out-of-plane forces prompts the question whether this modification of the coil geometry is actually advantageous or not. To this end, we notice that the tilting angle concept provides the possibility of choosing how to distribute the components of the force, so that the most convenient solution can be adopted to contrast them. As an example, had we optimized the coil by minimizing the normal force instead of the total force, we would have been able to totally eliminate the normal force, with the consequence of a small increase in the toroidal force.

3.2. TFC and plasma current

Next we introduce the plasma current, modeling it with a current loop. Since this simple choice does not include pressure effect, we have displaced the location of the loop outward 23 cm with respect to the torus axis (radius of the plasma current loop equal to $R_{\rm pl} = 1.55 > R_0 = 1.32$ m). This choice leads to shapes for the magnetic surfaces that better resemble those which would be obtained by employing an MHD equilibrium code, when conventional TFCs are considered. The optimization procedure carried out considering the magnetic field produced both by the TFCs (as done before) and the plasma ring (new component) leads to a reduction of the EM forces comparable to that obtained without plasma current.

3.3. Iterative procedure for self-consistent force-minimization including PFC and plasma current

So far the system TFC-plasma was optimized with the only goal of minimizing the forces acting on the TFCs. At flat-top, however, the plasma must be in toroidal equilibrium, i.e., the radially expanding force acting on the plasma, due to the plasma ring itself, must be balanced by an equal but opposed force. From a different point of view, the self-generated vertical field at the location of the plasma ring must be eliminated by an opposite field of equal magnitude produced with some other means. In a conventional tokamak, the latter field is provided by dedicated coils of the PFC system. In a TFC coil system which has been differentially-tilted all around its perimeter, however, the vertical field produced by the coils gives a contribution also at the plasma location. We have therefore added a full PFC system to our tokamak model, and repeated the optimization procedure to minimize the EM forces while at the same time keeping the plasma current in equilibrium. The poloidal coil field system has been modelled with a series of circular current-carrying loops, as shown in Fig. 4a for the case of conventional (untilted) TFCs.

For this case, we show in Fig. 4b a magnetic configuration in which the plasma current loop is kept in toroidal equilibrium by virtue of the vertical field produced by the PFC system. The total current flowing in the PFC system is equal to ~ 45 MA (note that not all the shown PFCs are energized), to be compared with the ~ 85 MA flowing in the TFC system that produces a toroidal field on axis of 13 T. The current flowing in the central solenoid produces a magnetic field inside the CS hole in the $+Z$ direction, in accord with the plasma current flowing in the $-\Phi$ direction and producing a magnetic field inside the CS hole in the $-Z$ direction, as it should be according to Faraday's induction law.

The optimization calculation in this case requires an iterative procedure to accomplish both goals of force minimization and plasma equilibrium. The resulting tilted angles and magnetic surfaces are presented in Fig. 5 [part (a) and (b), respectively].

As evident from part (b) of the figure, the current flowing in the outer rings of the PFCs has been reversed with respect to the reference configuration, as required to counterbalance the vertical field created by the differentially-tilted TFC system at the plasma location. To be more quantitative, the plasma current (11 MA) flows in the $-\hat{\boldsymbol{\Phi}}$ direction and generates a vertical field on the plasma itself equal to -3.67 T. At the end of the iterative procedure, the current in the PFC system is such to produce a vertical field at the plasma location equal to -3.56 T, while the differentially-tilted TFC system produces on the contrary a vertical field at the plasma location which goes in the $+\hat{\boldsymbol{Z}}$ direction. The sum of these three contributions gives zero vertical field at the plasma location, so that the plasma is in toroidal force balance.

The forces on the TFCs and due to the magnetic field produced by the TFCs, the PFCs and the plasma current are shown in Fig. 6a for the untilted system and Fig. 6b for the optimally differentially-tilted configuration. All results are normalized to 5.15×10^7 N/m, the maximum absolute value of the force in the untilted configuration.

From the figure we notice that the magnitude of the radial and vertical forces is notably reduced all over the coil, while the toroidal force has remained of the same order of magnitude (its maximum absolute value is about 10×10^6 N/m). All three components are subject to strong modulation, changing their signs more frequently than in the untilted case.

We end this section with two additional considerations. First, we observe that the vertical field produced by the differentially-tilted coil alone can in principle provide plasma toroidal balance. We have explored this interesting scenario by carrying out an optimization procedure subject to the constraint of plasma equilibrium without the aid of PFCs. The final configuration is characterized by wire tilting everywhere except for a region of the outer portion of the coils. Since this tilting introduces important modification to the magnetic flux surfaces, we do not report here these results, waiting to reconsider this scenario when the code will be coupled to a real MHD solver.

Second, we have also obtained a magnetic configuration that allows for the start-up of the plasma discharge. Keeping fixed the tilting of the TFCs presented in Fig. 5a, since we assume that force reduction is most important during flat-top operation, we modified the currents flowing in the PFCs so tho create a point of null inside the vacuum chamber. The goal has been achieved by modifying the current only in the last two PFCs, those located at $R = 2.19$ and 2.40 m.

4. D-shaped toroidal field system

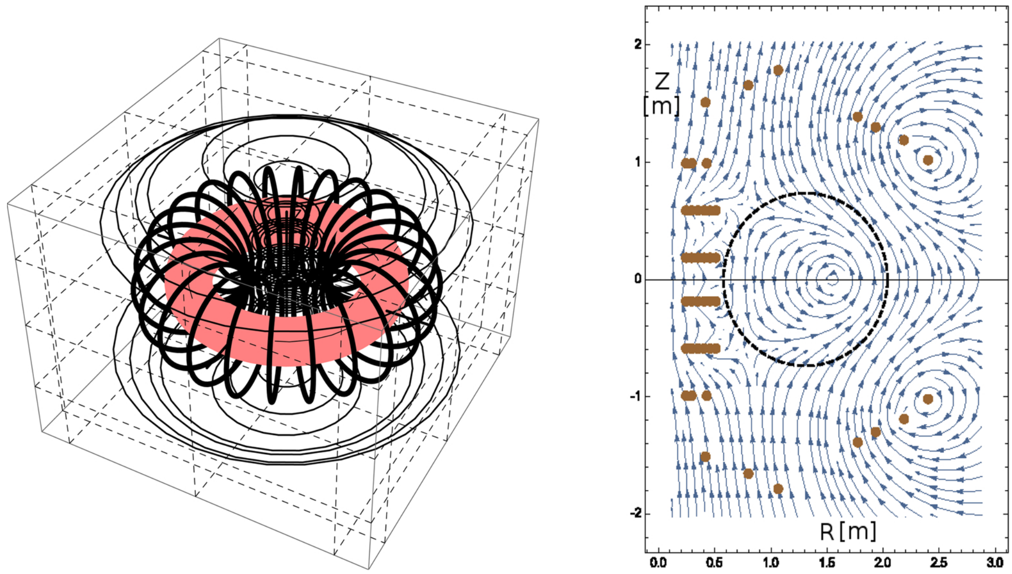
In this section we focus on D-shaped TFCs, being this kind of coils the one adopted by the majority of present and planned experiments, thanks to the favorable influence of this coil shape on the stability properties of the confined plasma.

We consider a tokamak with center location of the TFCs located at $R_{\rm center} = 1.32$ m, and we center the circular current loop mocking up the plasma current at $R_{\rm pl} = 1.50$ m. The shape of the coil is characterized by ellipticity and triangularity equal to, respectively, $\kappa_c = 1.44$ and $\delta_c = 0.344$. These parameters are proper of the project Ignitor, but well represent other proposals of compact high-field tokamaks. The PFC system is also derived from the Ignitor design but fewer coils are energized.

The optimization procedure is carried out only for inner leg of the coil. The reasons to put special emphasis on this case are manifold. First, we have found that D-shape coils do not allow for force reduction in the outer region of the coils (contrary to circular coils). Additionally, such a configuration is relevant both because this would be the simplest first step toward a tokamak which takes advantage of the tilting option, and because the possibility of obtaining a fusion-relevant plasma when a wider tilting is adopted needs to be verified by coupling the optimization code to an MHD equilibrium solver, so that the limitations intrinsic to the simple plasma current loop adopted in this work are overcome. The “central solenoid” associated with the differentially-tilted TFCs, visible Fig. 7a, has a radius of 0.58 m.

An iterative optimization run has produced tilting angles of 47.45° on average, with minimum and maximum angles equal to 42.25° and 57.67°. This variance about the mid-value is due to the influence of the field produced by the nearby current loops of the conventional CS. In Fig. 7b we trace the projections of the magnetic surfaces on the poloidal plane.

When the forces on the optimally differentially-tilted TFCs are compared with the corresponding forces on conventional (untilted) TFCs, see Fig. 8, it can be observed that the most obvious advantage of



(a) *Model of the conventional tokamak with circular TFCs.*

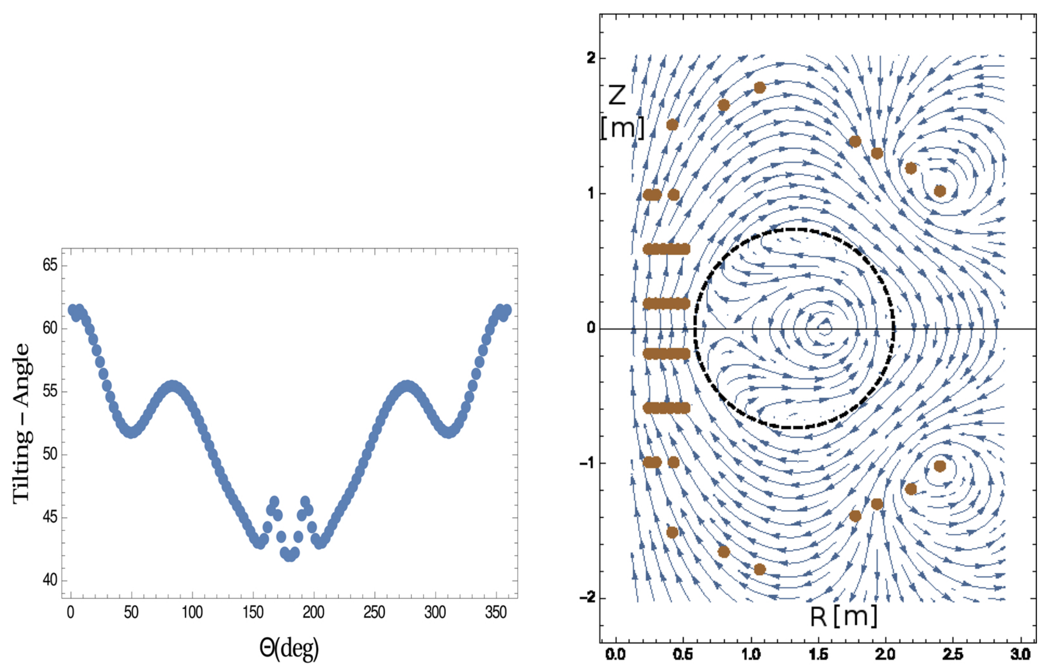
(b) *Poloidal projection of magnetic surfaces. Dots indicate the location of the PFCs.*

Fig. 4. Reference (untilted) configuration with circular TFCs. The plasma is in toroidal equilibrium. The dashed circle indicates the poloidal cross-section of the TFCs.

the inner leg tilting is a drastic reduction of the radial force acting on it (continuous line). The vertical force acting on the inner leg changes sign, and is subject to oscillations around values of the same order of magnitude as in the untilted case. Also the toroidal component remains of the same order of magnitude as in the untilted case, so that the tilting does not worsen the stress situation created by the poloidal field coils in conjunction with the plasma current.

In Table 1 we report the comparison of the poloidal flux generated by the conventional CS and the differentially-tilted TFC system. Even though the nature of the results obtained in this work is qualitative, mainly due to the lack of proper equilibrium calculations, these numbers indicate how the magnetic flux provided by the solenoid associated with the tilting is significant, thanks both to the larger dimension of the bore of the latter solenoid with respect to the conventional one ($R_{\text{in}} > R_{\text{CS}}$), and to the large angles of tilting needed for force minimization.

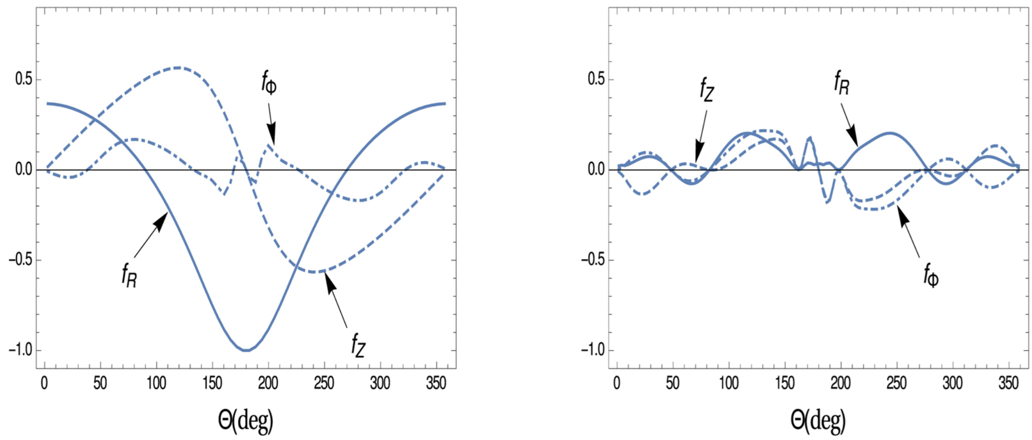
Before discussing possible operational scenarios with differentially-tilted coils, we go back to the statement presented in Section 2 that the



(a) *Optimal tilting angles*

(b) *Magnetic surfaces.*

Fig. 5. Optimized configuration with plasma current and PFCs at the end of the iterative procedure. The plasma ring, currying a current of 11 MA, is in toroidal equilibrium.



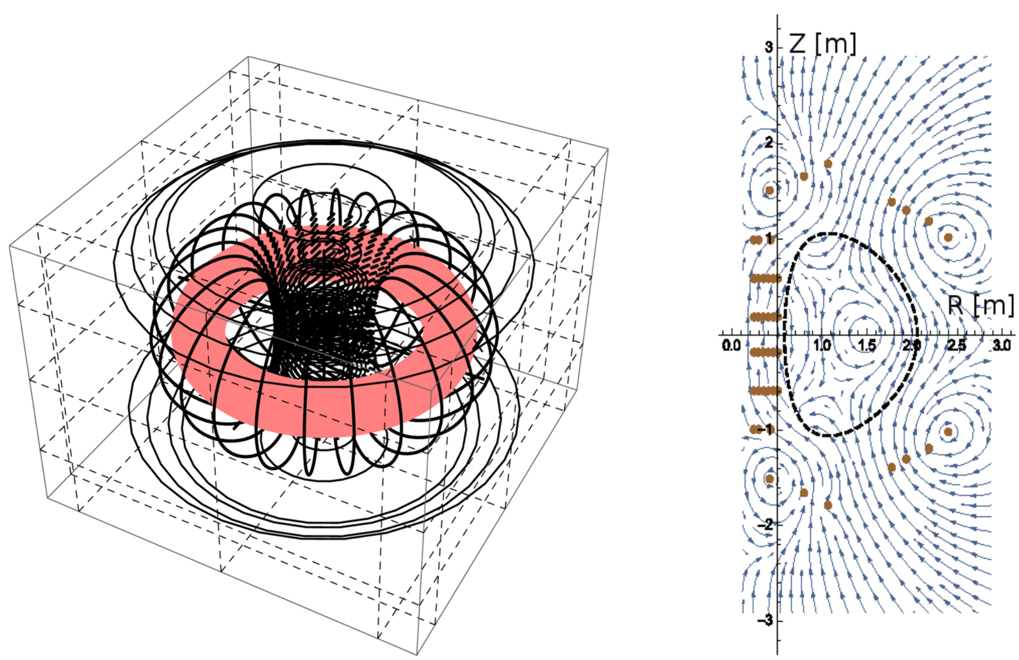
(a) *System with untilted TFCs.* (b) *System with differentially-tilted TFCs.*

Fig. 6. Comparison of forces per unit length acting on the TFCs and due to the field produced by the TFCs, PFCs and plasma current. In both cases the plasma is in radial equilibrium thanks to the PFC system. Forces are normalized to 5.15×10^7 N/m (maximum absolute value in the untilted configuration).

optimization procedure based on non-connected wires does not introduce errors which invalidate the conclusions on force reduction in a significant way. In Fig. 9 we present the continuous version of the optimized system shown in Fig. 7a, obtained by rotating appropriately in the azimuthal direction each single wire. Note how the coils, although each one is represented by a continuous line, do not form a closed system. To obtain the latter, it would be simply necessary to change the tilting angles of all wires of all coils by a small amount (in the example under consideration, about 0.075 degrees). We have evaluated the forces acting on the continuous coils, and computed the relative percentage difference with those obtained from the non-continuous coils. We have found that for most of the wires the difference ranges between a fraction to a few percent. There are very few wires, however, in which the difference is much higher. This is not surprising, and it simply means that in the continuous version of the coil the tilting angle of those few wires should be changed in order to achieve a comparable force reduction. We thus conclude that the optimization procedure based on discrete wires with fixed center-point, which brings benefits from the computational point of view, is valid, even though a more quantitative study of the differentially-tilted coil concept will have to rely on an upgraded version of the code which operates directly on continuous coils, and produces closed coil system as well. This code upgrade is underway.

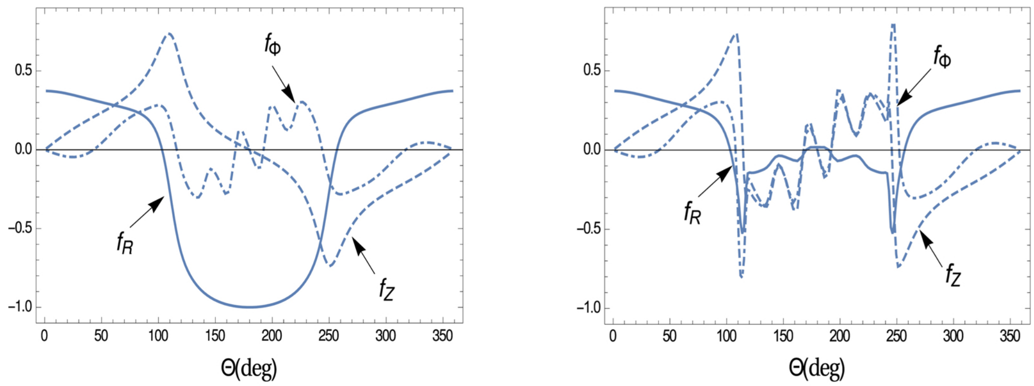
5. Operational scenario with differentially-tilted coils

We conclude by proposing a possible operational scenario for the configuration consisting of a TFC system with inner leg twisted plus a complete set of PFCs (CS plus equilibrium/shaping coils). The main two advantages associated with this configuration would be a reduced stress



(a) *Model of the tokamak with differentially-tilted inner leg.* (b) *Magnetic surface projections on the poloidal plane.*

Fig. 7. Configuration with differentially-tilted D-shaped TFCs and PFC system. The plasma is in toroidal equilibrium.



(a) *System with untilted TFCs.* (b) *System with differentially-tilted TFCs.*

Fig. 8. Comparison of forces per unit length acting on the TFCs. In both cases the plasma is in toroidal equilibrium thanks to the PFC system. Forces are normalized to 5.07×10^7 N/m (maximum value in the untilted configuration).

Table 1
Poloidal flux (in Wb) generated by the conventional CS and by the TFC system with differentially-tilted inner leg. The values inside the corresponding bores are reported in bold.

	CS	Differentially-tilted TFC
(0, R_{CS})	**11.41**	25.66
(0, R_{in})	11.58	**33.39**
(0, R_{pl})	6.45	22.89

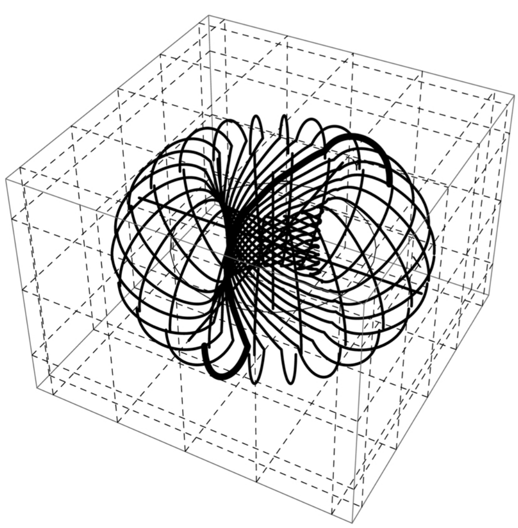


Fig. 9. Continuous version of the toroidal field coil system presented in Fig. 7a, with one coil highlighted with a thick line.

condition for the inner leg of the TFCs, and longer time of operation thanks to the additional magnetic flux produced by the twisted inner region of the TFC system. The scenario, sketched in Fig. 10, would take place according to the following steps. The sequence starts at time T_0 by linearly increasing the current flowing in the TFC system, I_{TFC}, thus producing a linearly increasing toroidal field in the plasma chamber which at this stage is still empty. The fuel gas is injected at time T_1. Since we expect that the rapidity of the increase in the I_{TFC} might very well be not sufficient to produce a large enough voltage for break-down, part of the flux swing produced by the conventional CS is utilized at this stage to ionize the fuel and initiate the plasma current (variant 1 in the figure). Alternatively, a brief burst of ECH could be used, thus saving the totality of CS flux for later deployment (variant 2 in the figure). At T_2 the task of inducing the plasma current is taken over by

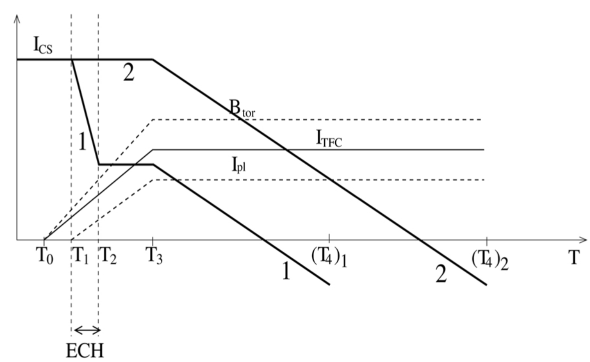


Fig. 10. Operational scenario for the differentially-tilted coil tokamak. Symbols: I_{CS} is the current in the conventional CS; I_{TFC} is the current in the TF coils; B_{tor} is the toroidal field; I_{pl} is the plasma current. Variant 1 employs the CS also for break-down, while in variant 2 the break-down is accomplished by a short burst of ECH.

the flux swing associated with the differentially-tilted TFCs, while the toroidal field keeps increasing. At T_3 the maximum allowed current in the TFCs is reached, and the conventional CS intervenes again to maintain the plasma current, as in conventional operation. The discharge is terminated at T_4 when the maximum negative value of I_{CS} is reached [$(T_4)_1$ for variant 1, and $(T_4)_2$ for variant 2]. During all phases of the discharge, the equilibrium and shaping coils of the PFC system intervene as needed, as in conventional discharges. We note that little experimental information is nowadays available regarding the formation and subsequent growth of a plasma while the toroidal field is increasing. The feasibility of such task has to be proven with ad-hoc experiments.

6. Summary and conclusions

The present work fits into the series of investigations performed by the fusion community regarding the potential benefits introduced by some form of tilting/twisting of the TFCs of a tokamak, investigations which are born theoretical but which have also led to relevant experimental results with the construction of various types of small-size tokamak prototypes. The contribution of our work, numerical in the approach, is based on the use of a dedicated code capable of performing flexible tilting optimization studies, in which the optimal tilting angle of any specified region of the coil, or along its entire perimeter, is obtained. Here, "optimal" means with respect to the minimization of the total (or a selected component of the) EM force acting on the coil. The main strategy adopted by the code is that of discretizing all coils in the system into a sequence of small straight wires, and computing the total field generated by them as a superposition of the field produced by each single wire. Although in all studies presented here we model the plasma current as a simple current loop, while a proper approach would require coupling the optimization code with an MHD solver, we believe that the present analysis represents a useful exploration of what are the potential advantages, and the problems as well, associated with differential tilting of TFCs.

We started our investigation by studying circular TFCs in isolation with the goal of minimizing the total force per unit length acting on each single wire. The optimization has lead to a configuration with notably reduced forces in the *R* and *Z* directions. The price to pay is, however, the insurgence of toroidal forces. Our optimization study indicates that the tilting angle concept provides the possibility of choosing how to distribute the components of the force, so that the most convenient solution can be adopted to contrast them.

Next we introduced the plasma current inside the TFC system, and addressed the problem of plasma toroidal equilibrium by including a PFC system. The optimized differentially-tilted configuration is characterized by a drastic reduction of the *R* and *Z* EM forces acting on the TFCs, while the toroidal component is not very different in magnitude from the conventional system. A start-up magnetic configuration with a point-of-null in a system with differentially-tilted TFCs is also possible.

As a successive step we considered D-shaped TFCs, presenting a configuration in which the tilting of the inner leg leads to a relevant reduction of the radial EM force. The differentially-tilted inner legs produce poloidal flux, the magnitude of which relative to that produced by the conventional CS has been quantified.

Operational scenario could be envisaged with differentially-tilted TFCs. Considering the configuration of D-shaped TFCs with differentially-tilted inner-leg plus a conventional PFC system, including the CS, we proposed a scenario in which the discharge is initiated by a combination of the conventional CS (or of an ECR heating device) and the solenoid formed by the twisted inner leg, and then maintained by the (remaining) CS flux-swing.

In conclusion, the differentially-tilted toroidal field coil concept appears to have the potential of improving current tokamak designs. The EM forces acting can be reduced so to be contrasted more efficiently, and the generated poloidal flux can be used to extend the duration of a discharge. Without arriving at the steady state reactor, a less stressed, longer pulse, higher field device can be conceived for operation at or near ignition. Before a more definitive conclusion on the usefulness of the differentially-tilted coil concept could be reached, however, further associated issues must be addressed. Among those, the question of plasma stability, as well as the adoption of more performing-shaped cross sections, a topic that has been only touched upon in this work. To accomplish this improvement, the optimization code must be coupled to an MHD solver, to remove the main limitation of the results presented in the present paper, i.e., that of modelling the plasma current with a simple current loop. Also, engineering aspects of the manufacturing of differentially-tilted coils must be defined. In principle, flexible, ribbon-like conductor could be adopted to generate the required configuration. Another important issue to be verified, related to the pulse length extension, is the compliance of the plasma to the simultaneous rise of the poloidal and toroidal magnetic fields, while increasing the plasma cross section. This point could be tested in existing experiments.

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