TITLE: DYNAMIC ONE-SIDED OUT-OF-PLANE BEHAVIOUR OF
UNREINFORCED-MASONRY WALL RESTRAINED BY ELASTO-PLASTIC TIE-RODS

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ABSTRACT

Past earthquakes have shown the high vulnerability of existing masonry buildings, particularly to out-of-plane local collapse mechanisms. Such mechanisms can be prevented if façades are restrained by tie rods improving the connections to perpendiculars walls. Whereas in the past only static models have been proposed, herein the non-linear equation of motion of a monolithic wall restrained by a tie rod is presented. The façade, resting on a foundation and adjacent to transverse walls, rotates only around one base pivot and has one degree of freedom. Its thickness is explicitly accounted for and the tie rod is modelled as a linear elastic - perfectly plastic spring, with limited displacement capacity. The model is used to investigate the response to
variations of wall geometry (height/thickness ratio, thickness), tie rod features (vertical position, length, prestress level), material characteristics (elastic modulus, ultimate elongation, yield strength) typical of historical iron. The most relevant parameter is the steel strength, whereas other characteristics play minor roles allowing to recommend reduced values for pre-tensioning forces. The force-based procedure customary in Italy for tie design is reasonably safe and involves protection also against collapse, although probably not enough as desirable.

**KEYWORDS**

rocking, historical iron, linear static procedure, prestress, steel, tie bar, ultimate elongation

**RUNNING HEAD**

Dynamic one-sided out-of-plane behaviour of unreinforced-masonry wall restrained by elastoplastic tie rods

1 **INTRODUCTION**

Earthquakes have shown that unreinforced-masonry structures frequently present a higher vulnerability than reinforced-concrete structures (Zucconi, Ferlito, and Sorrentino 2017; Zucconi, Sorrentino, and Ferlito 2017), with out-of-plane loading being particularly dangerous if connections of façades to transversal structures are inadequate (Bruneau 1994; Brando et al. 2018; Moon et al. 2014; Mendes et al. 2017). Metal tie rods are among the most ancient details adopted to improve earthquake performance in unreinforced masonry buildings (Figure 1a), and their use is documented in several countries, such as Haiti (Rosenboom, Kelley, and Paret 2014), Italy (Sorrentino, Brucoleri, and Antonini 2008; Lucibello et al. 2013; Gizzi et al. 2014), New Zealand (Campbell et al. 2012; Walsh et al. 2014; Marotta et al. 2015), and Turkey (Celik, Sesigur, and Cili 2009).
Steel ties have been recently proposed for cost-effective strengthening of both ordinary (Pomonis and Gaspari 2014) and monumental buildings (Degli Abbati et al. 2015). Shake table tests have proven their effectiveness in brickwork (Tomaževič, Lutman, and Weiss 1996) and natural stonework models (Magenes et al. 2014; Penna et al. 2016), provided that masonry disintegration does not occur (De Felice 2011; Liberatore et al. 2016). Vertical tendons and viscous dampers have been analytically investigated to control the response of rocking equipment and blocks (Makris and Zhang 2001; Dimitrakopoulos and DeJong 2012), and the use of superelastic alloys has been proposed for horizontal ties in the last decade (Indirli and Castellano 2008; Paret et al. 2008), but usually interventions are carried out with horizontal conventional-steel tie rods. Worked-out examples are available in books (Giuffrè 1993; Cangi, Caraboni, and De Maria 2010) and guidelines (Munari et al. 2010), but only static procedures are proposed without a proper validation by means of non-linear time-history analyses. Therefore, hereinafter a single-degree-of-freedom dynamic model of a monolithic wall restrained by a tie rod is presented. Although similar models have been proposed recently for a wall restrained at the top by a flexible diaphragm (Prajapati, AlShawa, and Sorrentino 2015; Giresini, Fragiacomo, and Lourenço 2015), the tie considered in the following is elasto-plastic, can fail if stretched beyond ultimate elongation and can be placed at any vertical position along the wall.
height. Moreover, its design is explicitly discussed and related parametric analyses are performed. The proposed model assumes that, as a result of a capacity design process, no failure at wall anchor (neither in the steel connection nor in the adjacent masonry) occurs and damage is concentrated in the easier-to-replace tie rod (Figure 1b). Finally, it is assumed that during motion no change in mechanism takes place, from single body to two bodies (Penner and Elwood 2016; Abrams et al. 2017), because the wall has a sufficiently low height/thickness ratio and because no openings are present that could further complicate the shape of the mechanism (Andreotti, Liberatore, and Sorrentino 2015; A1Shawa, Sorrentino, and Liberatore 2017).

2 ANALYTICAL MODEL

In this section the dynamic model of a monolithic wall of finite thickness, free to rotate on one side only and restrained by an elasto-plastic tie having limited displacement capacity, is presented (Figure 2a). The model has two sources of non-linearity: wall geometry, involving a lever arm changing with rotation (Figure 2b), and tie rod material, becoming plastic if a yield displacement is overcome. The tie can be positioned at any height of the wall, friction is assumed to be large enough to prevent sliding both at the base and at wall anchor.

![Figure 2. a) Geometrical parameters accounting for hinge indentation; b) One-sided displaced configuration ($\theta > 0$). c) Normalised self-weight restoring moment–rotation relationship.](image)

Considering only positive rotations, the analytical equation of motion is the following:
\[
\ddot{\theta} + \frac{p^2}{g} \left[ (\ddot{y}_g + g)U(\theta, \alpha, \alpha_i, \Delta_1, \Delta_2) - \dot{x}_g \cos(\alpha_i - \theta) + \frac{\epsilon \beta R_t}{m R_i} \cos(\alpha_t + \theta) \right] = 0
\] (1)

where \( \theta = \) wall rotation (Figure 2b) and dot indicating derivative with respect to time, \( p = \sqrt{m R_i g / I_O} = \) frequency parameter, \( m = \) mass of the wall, \( R_i = \) distance between centroid \( G \) and indented hinge \( O \), \( g = \) gravity acceleration, \( I_O = \) polar moment of inertia of the wall with respect to \( O \), \( \ddot{x}_g, \ddot{y}_g = \) horizontal, vertical ground motion acceleration, \( U = \) non-dimensional wall-self-weight restoring moment parameter equal to:

\[
U = \begin{cases} 
\frac{\theta}{\Delta_1 \alpha} \sin(\alpha_i - \Delta_2 \alpha) & \theta \leq \Delta_1 \alpha \\
\frac{\Delta_1 \alpha}{\sin(\alpha_i - \Delta_2 \alpha)} & \Delta_1 \alpha < \theta \leq \Delta_2 \alpha \\
\frac{\theta}{\sin(\alpha_i - \theta)} & \theta > \Delta_2 \alpha
\end{cases}
\] (2)

where \( \alpha = \arctan(B/H), \alpha_i = \) angle between \( R_i \) and vertical line through \( O \) (Figure 2a), and \( \Delta_1 \) and \( \Delta_2 \) non-dimensional parameters defining the three-branches law in Figure 2c, calibrated on experimental tests (Sorrentino, Alshawa, and Liberatore 2014). The hinge \( O \) is indented with respect to the geometric corner of the wall by a quantity, \( u \), depending on the masonry design compressive strength, \( f_{m,d} \), equal to:

\[
u = \frac{m g}{2 \cdot 0.85 f_{m,d} L_h}
\] (3)

where \( L_h = \) hinge length, coincident with the wall length if no openings are present. In Eq. (3) a stress block distribution of amplitude \( 0.85 \ f_{m,d} \) has been assumed, as customary in ultimate verifications. Nonetheless, linear distributions have been assumed in the literature for similar cases (Munari et al. 2010). As a consequence of Eq. (3), the following relation holds:

\[
\alpha_i = \arctan\left( \frac{B - 2u}{H} \right)
\] (4)
The tie is assumed to stay always horizontal although its wall anchor position $A$ (Figure 2b) is updated during the analysis, accounting for finite displacements. The tie contribution is determined by the following parameters: $F_y =$ yield force of the tie, $R_t =$ distance between wall anchor and $O$ (Figure 2a), $\chi =$ tie non-dimensional force (Figure 3), equal to

$$
\chi = \begin{cases} 
0 & \epsilon_t \leq \epsilon_r \\
1 + \frac{\epsilon_t - \epsilon_{\text{max}}}{\epsilon_y} & \epsilon_r < \epsilon_t \leq \epsilon_{\text{max}} \\
1 & \epsilon_{\text{max}} < \epsilon_t \leq \epsilon_u \\
0 & \epsilon_t > \epsilon_u 
\end{cases}
$$

$\epsilon_t =$ tie axial deformation, equal to:

$$
\epsilon_t = \frac{F_0}{F_y} + 2 \frac{R_t}{L_t} \cos\left(\alpha_t + \frac{\theta}{2}\right) \sin\frac{\theta}{2}
$$

where $F_0 =$ prestress force of the tie, $L_t =$ initial length of the tie, $\alpha_t =$ angle between $R_t$ and vertical line through $O$ (Figure 2a), $\epsilon_y =$ yield deformation of the steel, $\epsilon_{\text{max}} =$ maximum deformation reached so far in the time history, $\epsilon_r = \epsilon_{\text{max}} - \epsilon_y$, residual deformation reached so far in the time history, $\epsilon_u =$ ultimate deformation of the steel.

Figure 3. Tie cyclic non-dimensional force - deformation law.

The tie non-dimensional force, $\chi$, introduces an additional source of non-linearity because the tie is not active if a previous permanent deformation is not recovered or if it fails. When the tie fails a sudden release of elastic potential energy occurs. If the wall anchor is bilaterally connected to the façade and the failure occurs close to the anchor, so that no whiplash effect is
involved, the potential energy transforms itself in a kinetic energy contribution that gives to the wall an additional angular velocity, $\dot{\theta}_f$, equal to:

$$\dot{\theta}_f = f_{y,d} \frac{A_l L_t}{E_s I_o}$$

(7)

where $f_{y,d}$ = steel yield design strength, $A_l$ = tie cross section area, $E_s$ = steel Young’s modulus.

Because wall anchors are usually just adjacent to the façade, involving a monolateral connection, and the tie failure occurs frequently away from the wall, hereinafter when the tie fails its contribution will be neglected without modifying the wall angular velocity.

During the time history, when the rotation becomes zero the wall hits the base and the transversal structures. At this time an energy dissipation occurs, by means of a negative velocity reduction coefficient. The minus sign involves a rebound, hence keeping rotations positive. The value of the velocity reduction coefficient, also known as coefficient of restitution, has been determined following the conservation of angular momentum approach proposed by Housner (1963) for two-sided rocking, and for one-sided rocking is equal to:

$$e_{an,1s} = 1.05 \left(1 - 2 \frac{m R_i^2}{I_o} \sin^2 \alpha_i \right) \left(1 - 2 \frac{m R_i^2}{I_o} \cos^2 \alpha_i \right)$$

(8)

valid for a generic geometry. The 1.05 coefficient was experimentally calibrated in Sorrentino, AlShawa, and Decanini (2011).

3 PARAMETRIC ANALYSIS

3.1 Tie code design

As customary in the Italian technical literature (Giuffrè 1993; Cangi, Caraboni, and De Maria 2010; Munari et al. 2010), the tie is designed according to a force-based procedure lately based
on the Commentary to the Italian Building Code (CMIT 2009), so as to have the following horizontal collapse load multiplier:

\[ \alpha_0 = \frac{a_g S CF e^*}{q} \]  

(9)

where \( a_g \) = maximum ground acceleration expected for the life safety limit state in a horizontal rock site, \( S \) = site response coefficient (depending on topographic and stratigraphic conditions), \( CF \) = confidence factor, \( e^* \) = participating mass factor (assumed equal to 1 in the case of a monolithic wall, according to the Italian procedure (Sorrentino et al. 2017)), \( q \) = behaviour factor (assumed equal to 2 according to the Commentary to the Italian Building Code (CMIT 2009)).

The tie force, \( F_y \), granting the load multiplier \( \alpha_0 \) is equal to

\[ F_y = \frac{mg(\alpha_0 H - B + 2u)}{2H_t} \]  

(10)

where \( H_t \) = tie force lever arm at rest (Figure 2a). The consequent tie cross section area, \( A_t \), is equal to:

\[ A_t = \frac{F_y}{f_{yd}} \]  

(11)

In the following, yield design strength has been computed for a partial safety factor larger than one, because the tie is a new element. However, in the technical literature other authors assume a unity value (Munari et al. 2010; Cangi, Caraboni, and De Maria 2010). This issue can be properly resolved only within a fully probabilistic calibration analysis, which is outside the scope of this paper.

In a real case design, the cross section area \( A_t \) would be rounded up to account for commercial rod diameters. However, such rounding up would involve an unsystematic effect on the parametric analyses performed hereinafter, also because the assumed unity wall length implicates small
tie rod section areas. Therefore, in the following, the cross section area has been simply computed according to Eq. (11).

Code design has been always performed assuming a modern steel S235 as specified in the Italian Building Code (DMI 2008) or in Eurocode 3 (EC3-1-1 2005), and with all relevant values summarised in Table 1. Once the geometry of the tie rod was defined, only a single investigated parameter (pertaining masonry geometry, tie-rod geometry and steel mechanical characteristics) has been changed according to the ranges specified in the following sections, whereas all other parameters are kept constant for all analyses.

Table 1. Assumed values for tie code design

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit of measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earthquake return period</td>
<td>$T_R$</td>
<td>500</td>
<td>years</td>
</tr>
<tr>
<td>Peak ground acceleration on stiff ground type</td>
<td>$a_g$</td>
<td>0.26</td>
<td>g</td>
</tr>
<tr>
<td>Site response coefficient</td>
<td>$S$</td>
<td>1.33</td>
<td></td>
</tr>
<tr>
<td>Confidence factor</td>
<td>$CF$</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>Modal participation factor</td>
<td>$e^*$</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Masonry bulk specific weight</td>
<td>$w$</td>
<td>20</td>
<td>kN/m$^3$</td>
</tr>
<tr>
<td>Masonry design compressive strength</td>
<td>$f_{m,d}$</td>
<td>125</td>
<td>N/cm$^2$</td>
</tr>
<tr>
<td>Wall length</td>
<td>$L$</td>
<td>1.0</td>
<td>m</td>
</tr>
<tr>
<td>Tie rod normalised length</td>
<td>$L_t / B$</td>
<td>10</td>
<td>-</td>
</tr>
<tr>
<td>Tie rod normalised prestress force</td>
<td>$F_0 / F_y$</td>
<td>0.1</td>
<td>-</td>
</tr>
<tr>
<td>Steel Young’s modulus</td>
<td>$E_s$</td>
<td>210</td>
<td>GPa</td>
</tr>
<tr>
<td>Steel partial safety coefficient</td>
<td>$\gamma_{m0}$</td>
<td>1.05</td>
<td>-</td>
</tr>
<tr>
<td>Steel characteristic yield strength</td>
<td>$f_{y,k}$</td>
<td>235</td>
<td>MPa</td>
</tr>
<tr>
<td>Steel ultimate elongation</td>
<td>$\varepsilon_u$</td>
<td>0.20</td>
<td>-</td>
</tr>
</tbody>
</table>
3.2 Historical tie characteristics

In the previous section the tie has been designed assuming a modern steel, which may have different strengths but usually have unique elastic modulus and ultimate elongation. On the contrary, historical ties display rather different properties, as determined by Calderini et al. (2016). Based on experimental tests performed on a set of tie rods recovered from restoration works or building demolitions, the authors present mean and standard deviation of elastic modulus, yield strength, and ultimate elongation summarised in Table 2. Such values will be the base for the parametric analyses developed in Sect. 4.2 and Sect. 4.3.

Table 2. Mean and standard deviation of Young’s modulus, yield strength and ultimate elongation of historical tie rods (Calderini et al. 2016)

<table>
<thead>
<tr>
<th></th>
<th>$E_s$</th>
<th>$f_y$</th>
<th>$\varepsilon_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GPa</td>
<td>MPa</td>
<td>-</td>
</tr>
<tr>
<td>$\mu$</td>
<td>209</td>
<td>218</td>
<td>0.169</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>76.1</td>
<td>46.1</td>
<td>0.091</td>
</tr>
</tbody>
</table>

3.3 Ground motion records

The dynamic model defined in Sect. 2 is investigated under natural records consistent with site-specific and return-period-specific spectral shapes. The site considered is L’Aquila, Southern Italy, for a ground type C according to Eurocode 8 (EC8-1 2004). Three event return periods have been considered: 50, 500 and 1000 years, approximately corresponding to damage limitation, life safety and collapse prevention limit states of ordinary buildings (DMI 2008). The 500 years return period is considered as the most significant and systematically investigated, whereas the 50 and 1000 years return periods are taken into account for comparison purposes only.

In order to increase efficiency and sufficiency of record selection for a system that has an amplitude-dependant vibration period, spectral compliance is pursued in terms of log-average spectral acceleration over the period range 0.1-2.0 s (Kohrangi et al. 2017). Three sets of 40
records each have been used, entailing different accelerograms selected within the RINTC pro-
ject (Iervolino, Spillatura, and Bazzurro 2017).

The peak ground acceleration and Housner Intensity (Housner 1952) of the records are pre-
sented in Figure 4. Peak ground acceleration is relevant for mechanism activation (Housner
1963), whereas Housner Intensity is well correlated with mechanism failure (Marotta et al.
2018). Given the asymmetry of the mechanism under consideration, with rotation allowed only
on one side due to the presence of transversal structures, the records are considered with both
positive and negative polarity, thus obtaining 80 records.

4 ANALYSES RESULTS

4.1 Tie effectiveness and role of wall geometry

Role of wall geometry is emphasised in Figure 5. Four walls have been selected, having
height/thickness ratio equal to 8 and 12, and thickness equal to 0.6 and 0.9 m. Therefore, it will
be possible to observe the role of aspect ratio as well as that of scale. Geometry has been se-
lected using mean, $\mu$, and standard deviation, $\sigma$, values of a population of about 300 masonry
walls belonging to ordinary buildings surveyed in Abruzzi region in Southern Italy (Sorrentino
2014), where the wall may span over several floors and the thickness is that at ground floor.
Height/thickness ratio values are approximately equal to corresponding $\mu - \sigma$ and $\mu$ values,
respectively. No larger values have been assumed because they may involve a change of mech-
anism, from one-body cantilever wall to two-body vertical spanning wall (Penner and Elwood
2016; Abrams et al. 2017). Wall thickness values are approximately equal to corresponding $\mu$
$- \sigma$ and $\mu + \sigma$ values respectively, thus covering a sufficiently ample range.
Figure 4. Peak ground acceleration and Housner Intensity of selected records. Return period of the earthquake = a-b) 50 years, c-d) 500 years, e-f) 1000 years.

In Figure 5, the normalised maximum rotation, $\theta_{\text{max}} / \alpha$, of a façade with no tie rod and with tie rod varying its normalised height, $H_r / H$ (Figure 2a) is plotted. Each marker in the subplots is related to one of the assumed 80 records. For each response it is annotated whether the tie remains elastic or becomes plastic. No failure of the tie has been observed, thus proving the code approach to be safe. The median, the mean and the 90$^{\text{th}}$ percentile of the 80 time histories normalised maximum rotations are computed. In the case of the wall without tie rods, median
normalised maximum rotation varies between 0.24 and 0.68, whereas mean value varies between 0.31 and 0.67. Hence, a skewness is present in the sample and computing the mean is on the safe side. The range of values is rather ample as an effect of aspect ratio (the lower the height/thickness ratio the smaller the rotation) and scale (the larger the thickness the smaller the rotation). The 90\textsuperscript{th} percentile varies between 0.61 and 1.00.

If similar statistics are computed for the tied walls, much lower values can be observed. Median varies 0.08 and 0.16, mean between 0.12 and 0.20, 90\textsuperscript{th} percentile between 0.27 and 0.49. Hence, ties are rather effective, confirming what found in static analyses by Casapulla et al. (2016). It is worth mentioning that, as an effect of the intervention, the range of variation is much smaller than for the unrestrained wall but wall geometry still plays a role despite tie presence. On one hand, the effect of aspect ratio is negligible: two walls having the same size and different height/thickness ratios have similar responses (in Figure 5 compare a with b, c with d). Although the response of untied walls is influenced by aspect ratio, the analyses show that in tied walls tie cross section (larger for more slender walls) compensates the effect of aspect ratio. On the other hand, walls of same height/thickness ratio but different size experience different maximum rotations (in Figure 5 compare a with c, b with d), a point further discussed in Sect. 4.4. Finally, the height position of the tie rod, thanks to compensating cross section area, has a negligible effect on all considered statistics but the role of tie characteristics is discussed in detail in the following section.
4.2 Role of tie characteristics

The role of tie geometry can be investigated in Figure 6, where it is summarised in terms of 90th percentile of maximum normalised rotation. If the size of the cross section area is designed according to the code, hence varying with tie rod position along the wall height ($H_t / H$), the maximum rotation experienced by the wall is similar in each considered wall and all ties become plastic, a condition emphasised with a letter P above each bar of the plot. In the same Figure, the role of the non-dimensional length of the tie rod, $L_t / B$, has been investigated. No
survey data were found about such parameter, hence values between 4 and 20 have been assumed as rather wide but still reasonable.

The tie rod length influences the tie rod stiffness and the ultimate elongation. However, no tie rod fails, hence only the effect on stiffness is relevant in the investigated wall geometries. Shorter tie rods are stiffer but present a smaller yield displacement and become plastic before longer ties (Figure 7 - Figure 8). During load reversal the tie is inactive until the permanent deformation is not recovered, and the wall can be substantially rotated when the next significant ground motion pulse occurs. Hence, shorter tie rods tend to be associated with slightly larger maximum rotations. Nonetheless, the overall effect of tie rod length is rather limited.
Figure 7. Role of normalised length, $L_t/B = 10.0$. a) time history of normalised rotation, b) time history of tie normalised axial force, c) normalised axial force – elongation. Plot c) starts from prestress elongation.

Figure 8. Role of normalised length, $L_t/B = 4.0$. a) time history of normalised rotation, b) time history of tie normalised axial force, c) normalised axial force – elongation.

In Figure 9 the role of prestress force is investigated. In the technical literature there are no established recommended values. Dolce et al. (2006) suggest a tensile prestress in the cross section approximately equal to 10 MPa. Podestà (2012) presents design examples with a prestress in the range 38-113 MPa. Lagomarsino and Calderini (2005) found in the ties of three buildings values in the 32-129 MPa range, which would involve a prestress normalised over average yield strength in Calderini et al. (2016) in the range 0.15-0.59. Rainieri et al. (2015) found in another building normalised prestress in the range 0.16-0.23. Consequently, for the parametric analysis a reasonable range 0.0-0.6 was assumed, comprising the lack of any pre-
stress as well as rather large forces at wall anchor. The overall effect of prestress force is negligible and without systematic trends, even though a normalised prestress as large as 0.80 is assumed (not shown for the sake of brevity). This behaviour can be explained with all ties becoming plastic. However, for 50 years return period earthquakes (corresponding to damage limitation state of ordinary buildings) the ties remain elastic, a condition emphasised with a letter E above each bar of the plot, and a higher prestress reduces rotation amplitude (Figure 10). Nonetheless, differences are rather small and the use of large prestress forces, possibly involving damage at wall anchor already when the intervention is carried out, is questionable. Therefore, the field investigation of the tie rod current stress state seems to be of limited interest.

Figure 9. Role of normalised prestress force, $F_0/F_y$. Response in terms of the 90th percentile of normalised maximum rotation, $\theta_{\text{max}}/\alpha_i$, varying tie rod normalised height, $H_t/H$ (Figure 2a), and wall geometry (a-d). Plastic response of the tie emphasised with a letter P above the bars in the plot. Assumed values for tie code design as in Table 1.
4.3 **Role of steel characteristics**

The role of Young’s modulus is analysed in Figure 11, wherein the range of values is that derived from mean, $\mu$, and standard deviation, $\sigma$, values in Table 2, thus assuming $E_s = \mu \pm 1.64\sigma$, $\mu \pm \sigma$, $\mu$. It is worth mentioning that the mean value of this range is rather similar to the codified value assumed for the design tie rod in Table 1. The effect of an increased Young’s modulus is limited and similar to that of a shorter tie, already discussed in Figure 6. Hence, an increased maximum rotation can be observed, at least at life safety limit state. At damage limitation limit state no tie becomes plastic, hence the stiffer tie rod involves smaller rotations but again with rather negligible differences (not shown for the sake of conciseness). Similar results, again not shown for the sake of brevity, were obtained varying the tie rod axial stiffness.
Yield strength is studied in Figure 12, wherein varies according to mean and standard deviation values in Table 2, thus assuming $f_{y,d} = \mu \pm 1.64\sigma$, $\mu \pm \sigma$, $\mu$, with mean value rather close to the codified design value. It is important to reiterate that the ties are designed according to the code procedure described in Sect. 3.2, hence walls of given geometry and tie-rod position share the same tie-rod cross section, same Young’s modulus, same prestress force, as well as all other parameters, and in the parametric analyses only yield strength varies. Consequently, yield force and yield displacement increase with increasing yield strength and, conversely, experienced maximum rotation reduces. All this considered, yield strength is the most relevant parameter and influences the response, as one could expect. Steel strength can be estimated by means of correlation with hardness tests (Gaško and Rosenberg 2011; Pavlina and Vantyne 2008), for
which field procedures have been proposed (Haggag 2001; Mehdianpour and Waßmuth 2016), although so far not on tie rods.

Figure 12. Role of yield strength, $f_y$. Response in terms of the 90th percentile of normalised maximum rotation, $\theta_{\text{max}}/\alpha_i$, varying tie rod normalised height, $H_t/H$ (Figure 2a), and wall geometry (a-d). Plastic response of the tie emphasised with a letter P above the bars in the plot. Assumed values for tie code design as in Table 1.

Considering the great relevance that displacement-based procedures have gained in earthquake engineering it is worth studying the role of the ultimate elongation (Figure 13). This parameter varies according to mean and standard deviation values in Table 2, thus assuming $\varepsilon_u = \mu \pm 1.64\sigma, \mu \pm \sigma, \mu$, with mean value rather close to the codified design value. Surprisingly, there is a very limited effect of ultimate elongation on the response and tie failures, a condition emphasised with a letter F above relevant bars of the plot, occur only for very low (and unlikely)
ultimate elongation values. Of course, ultimate displacement is a function of ultimate elongation and tie rod initial length that, because normalised by wall thickness, involves a higher relevance of ultimate elongation for thinner walls and for rods located at higher positions where displacement is larger. All this considered, the analyses have been repeated halving the normalised length \( \frac{L_t}{B} = 5 \) but the differences are limited (Figure 14). Only very short normalised length \( \frac{L_t}{B} = 2 \), not shown for the sake of conciseness) can induce some failures for an elongation equal to 0.08, which is still a rather conservative value within historical values although approximately equal to codified elongation of concrete rebars (DMI 2008). Hence, at least for investigated wall geometries and tie rod lengths, the ultimate elongation is not a crucial parameter and force-based design is acceptable.
Figure 13. Role of ultimate elongation, $\varepsilon_u$. Response in terms of the 90th percentile of normalised maximum rotation, $\theta_{\text{max}}/\alpha_i$, varying tie rod normalised height, $H_t/H$ (Figure 2a), and wall geometry (a-d). Plastic or failed response of the tie emphasised with a letter P or F, respectively, above the bars in the plot. Assumed values for tie code design as in Table 1. Normalised tie rod length $L_t/B = 10$. 

a) $H/B = 8, B = 0.6 \text{ m}$  

b) $H/B = 12, B = 0.6 \text{ m}$  

c) $H/B = 8, B = 0.9 \text{ m}$  

d) $H/B = 12, B = 0.9 \text{ m}$
Figure 14. Role of ultimate elongation, $\varepsilon_u$. Response in terms of the 90th percentile of normalised maximum rotation, $\theta_{\text{max}}/\alpha_i$, varying tie rod normalised height, $H_t/H$ (Figure 2a), and wall geometry (a-d). Plastic or failed response of the tie emphasised with a letter P or F, respectively, above the bars in the plot. Assumed values for tie code design as in Table 1. Normalised tie rod length $L_t/B = 5$.

4.4 Code considerations

In Figure 5 it was shown that a force-based tie design according to the Commentary to the Italian Building Code (CMIT 2009) is rather effective when compared to the same walls without ties. According to the Commentary, at life safety limit state (static) demand / capacity ratio should not exceed 0.40, unless horizontal-structures collapse occurs for smaller displacements. It was shown in Figure 5 that median, over 80 time histories, of normalised maximum rotation varies between 0.08 and 0.16, mean between 0.12 and 0.20, 90th percentile between 0.27 and 0.49. Hence, code recommendations are certainly met.
Nonetheless, variation in safety level is substantial because the force-design procedure described in Sect. 3.1, is not capable to account for the size of the wall, the well-known scale effect (Housner 1963). This behaviour is related to the different contribution in terms of potential energy that the tie designed according to strength can deliver to walls of same height/thickness ratio but different size.

Assimilating for the sake of brevity the wall without tie rod to a rectangular block, the potential energy with respect to centroid position is equal to:

\[ V_W = mgR_i[cos(\alpha_i - \theta) - cos\alpha_i] \]  

(12)

Excluding previous plastic cycles, the potential energy of the tie rod is equal to:

\[ V_{TR} = \frac{1}{2}E_sA_tL_t \begin{cases} \frac{\varepsilon_t^2}{\varepsilon_y^2} & \varepsilon_t \leq \varepsilon_y \\ \frac{\varepsilon_y^2}{\varepsilon_y^2} & \varepsilon_y < \varepsilon_t \leq \varepsilon_u \\ 0 & \varepsilon_t > \varepsilon_u \end{cases} \]  

(13)

In Figure 15 the potential energy of the wall restrained by tie rods is normalised by that of the unrestrained wall, in order to emphasise the role of the tie. It is evident that walls having the same aspect ratio have almost coincident potential ratio, irrespective of the tie rod vertical position. On the contrary, if the size of the wall is changed a difference can be observed in the potential energy ratio. Hence, alternative procedures such as displacement- or energy-based (Sorrentino et al. 2017) should be explored, although they may involve a more convoluted design process for the practitioners.
The previous analyses have been repeated assuming an earthquake with return period $T_R = 1000$ years that, according to the Italian building code (DMI 2008), can be approximately related to the collapse prevention limit state of an ordinary building (Figure 16). For the unrestrained wall the median of the normalised maximum rotation varies between 0.74 and 1.00, mean between 0.69 and 0.92, 90th percentile is equal to 1.00 (overturning) for all geometries. It is worth mentioning that no value larger than 1.00 (overturning) is possible by default, thus explaining why median is larger than mean. For the tied walls the median varies between 0.35 and 0.66, mean between 0.40 and 0.70, 90th percentile between 0.70 and 1.25. In this case the 1.00 threshold is exceeded because of the presence of an elongated tie. Reported statistics should be compared with a larger normalised rotation than for the life safety limit state. An Eurocode draft under preparation (Lu et al. 2016) assumes for the collapse prevention limit state a (static) $\theta_{\text{max}} / \alpha_i = 0.60$. Hence, current code procedure seems to involve a reasonable protection against collapse, although probably not as large as desirable. Failures of tie rods can be observed in some instances and a conventional normalised maximum rotation equal to unity has been plotted. Contrary to Figure 13 and Figure 14, in these plots tie failure always involves

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**Figure 15.** Potential energy of the wall restrained by tie rods ($V_w + V_{TR}$), normalised by that of the unrestrained wall ($V_w$) varying wall geometry and hinge position. No previous plastic cycle.
wall overturning, therefore pushover analyses (Lagomarsino 2015; Cangi, Caraboni, and De Maria 2010; Podestà 2012) should probably neglect the curve beyond tie crisis.

Figure 16. Normalised maximum rotation, $\frac{\theta_{\text{max}}}{\alpha_i}$, of a façade with no tie rod and with tie rod varying its normalised height, $H/H_f$ (Figure 2a), and wall geometry (a-d). Assumed values for tie code design as in Table 1. Each marker related to one of the assumed 80 records (40 records with positive and negative polarity). Plastic or failed response of the tie emphasised. Failed response involves overturning, with a conventional normalised maximum rotation equal to unity. Record selection according to an earthquake return period $T_R = 1000$ years.

The design performed in Sect. 3.1 assumed a behaviour factor $q = 2$ in Eq. (9), as recommended in the Italian procedure (CMIT 2009). In Figure 17 analyses are repeated, for 500 years earthquake records, assuming for design $q = 1$. Median of normalised maximum rotation becomes now as low as 0.01-0.02, mean 0.03-0.04, 90th percentile 0.09-0.15. Hence, such behaviour factor involves rather conservative results. In Figure 18 analyses are repeated assuming for design $q = 3$. Median of normalised maximum rotation becomes now as high as 0.17-0.31, mean
0.21-0.38, 90\textsuperscript{th} percentile 0.41-0.81. For this behaviour factor failures of tie rods can be observed even for 500 years return period earthquake records, which is probably unacceptable.

Figure 17. Normalised maximum rotation, \( \theta_{\text{max}}/\alpha_i \), of a façade with no tie rod and with tie rod varying its normalised height, \( H_t/H \) (Figure 2a), and wall geometry (a-d). Each marker related to one of the assumed 80 records (40 records with positive and negative polarity). Elastic or plastic response of the tie emphasised. Assumed values for tie code design as in Table 1, with the exception of behaviour factor \( q = 1 \).
Figure 18. Normalised maximum rotation, $\theta_{\text{max}}/\alpha_i$, of a façade with no tie rod and with tie rod varying its normalised height, $H_t/H$ (Figure 2a), and wall geometry (a-d). Each marker related to one of the assumed 80 records (40 records with positive and negative polarity). Elastic, plastic or failed response of the tie emphasised. Failed response involves overturning, with a conventional normalised maximum rotation equal to unity. Assumed values for tie code design as in Table 1, with the exception of behaviour factor $q = 3$.

5 CONCLUSIONS

In this paper the non-linear equation of motion of a single-body wall restrained by an elastoplastic tie rod of finite displacement capacity is proposed. The wall has a flat base and an indented corner pivot about which it can rotate on one side only. The response has two sources of non-linearity, one geometry related and one material related. The material related non-linearity is further complicated by the tie being inactive if a previous permanent deformation is not recovered or if the tie fails.
The tie rod is designed according to the Commentary to the Italian Building Code, following a force-based approach and assuming a modern steel, and the restrained wall is excited by a set natural records. The intervention is rather effective in reducing the wall initial vulnerability, whatever the vertical position of the tie. The code procedure assumes a behaviour factor $q = 2$, a value that guarantees substantial protection both at life safety limit state and some protection at collapse prevention limit state. On the contrary, a unity value would involve a rather conservative design and $q = 3$ induces larger rotations and a few failures already for 500 years return period earthquakes, failures that are not present for lower values of behaviour factor. Unfortunately, the force-based procedure is unable to capture the different safety levels of two walls of same height/thickness ratio but different size. Therefore, in the future displacement- or energy-based procedures could be investigated.

Tie rods are not a recent technique because they were used extensively in the past. Therefore, it is useful to investigate whether historical iron/steel can prove as effective as modern steel by means of a parametric analysis. First of all, tie rod geometry has been defined according to a modern design, then a single parameter at time has been changed. Meaningful ranges of iron/steel characteristics are assumed from experimental literature on historical specimens.

The role of Young’s modulus is very limited both at life safety and damage limitation limit states and the same applies to the tie rod length, which affects stiffness. Similarly, ultimate elongation is relevant only if very low and unlikely values are assumed. Therefore, force-based procedures prove to be at the same time effective and easy to implement. Yield strength is much more relevant and specific non-destructive techniques, possibly based on measurement of hardness, should be specifically developed for steel tie rods. Non-destructive techniques already exist to estimate prestress that, however, shows little relevance for both ultimate and serviceability limit states. Therefore, large pretensioning forces should be avoided in new interventions, in order to avert masonry damage at wall anchor. Wall anchor failures and the formation of
intermediate hinge were excluded in this analysis, but need to be properly investigated in future studies.

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