Synchrotron resonant radiation from nonlinear self-accelerating pulses

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Abstract: Solitons and nonlinear waves emit resonant radiation in the presence of perturbations. This effect is relevant for nonlinear fiber optics, supercontinuum generation, rogue waves, and complex nonlinear dynamics. However, resonant radiation is narrowband, and the challenge is finding novel ways to generate and tailor broadband spectra. We theoretically predict that nonlinear self-accelerated pulses emit a novel form of synchrotron radiation that is extremely broadband and controllable. We develop an analytic theory and confirm the results by numerical analysis. This new form of supercontinuum generation can be highly engineered by shaping the trajectory of the nonlinear self-accelerated pulses. Our results may find applications in novel highly efficient classical and quantum sources for spectroscopy, biophysics, security, and metrology.

OCIS codes: (190.0190) Nonlinear optics; (190.4370) Nonlinear optics, fibers; (320.5540) Pulse shaping; (350.5610) Radiation.

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1. Introduction

Finite energy Airy pulses are non-spreading electromagnetic expressed as a truncated Airy function [1]. They are the temporal counterpart of finite energy Airy beam [1,2], firstly introduced in the context of quantum mechanics [3]. Similar to Airy beams, Airy pulses also display truly remarkable properties such as quasi-nondispersive evolution, self-reconstruction and self-acceleration [1, 2, 4]. Compared to the self-bending trajectory of Airy beams, Airy pulses exhibit self-accelerating or self-decelerating dynamics [5], and have asymmetric temporal profile with rapidly oscillating tails due to the cubic phase modulation. Various authors have shown that Airy pulses have many exciting applications, including linear light bullets generation [6, 7], supercontinuum generation [8], self-focusing dynamics [9, 10], manipulation of Raman-induced frequency effects [11–13], optimizing laser-cell membrane interactions [14], laser processing [15], and more.

Self-accelerating Airy wave-packets only exist in linear media. Wherefore, nonlinear propagation effects give rise to a severe distortion of the Airy profile, as soliton shedding [16]. Notably, temporal self-accelerating solitons with similar Airy-like profile in nonlinear Kerr media were reported by Giannini *et al.* as early as 1989 [17]. Recently, this concept has been used to find spatial self-accelerating solitons from Kerr nonlinear media [18, 19] to nonlocal nonlinearity [20] as well as a quadratic response nonlinear media [18, 21]. Interestingly, linear Airy beam in nonlinear media evolves into nonlinear self-accelerating solitons [22].

Resonant radiation (RR) - or optical Cherenkov or dispersive wave radiation - is a ubiquitous nonlinear optical process, which originates from a stable temporal soliton propagation with additional higher-order dispersions [23, 24]. RRs is extremely relevant in the context of supercontinuum generation and frequency combs [25, 26], and an effective wavelength conversion technique in the deep and vacuum ultraviolet [27, 28], visible [29] and mid-infrared [30, 31]. These exciting results stimulate an increasing interest on RRs, leading to new ideas as negative frequency [32] and diffractive [33] RRs as well as backward [34] and super [35] RRs. Various authors studied the control of RRs by using multimode fibers [36] and dispersion oscillating fibers [37], and a new type of collapse-arrested mechanism via RRs [38].

Previous investigations on RRs considered pulses with symmetric profiles as Gaussian and hyperbolic secant pulses. More recently, RRs emitted from a self-accelerating wave-packets have been also reported [39]. However, the investigation is restricted to the linear Airy pulse. An open question concerns the dynamics of RRs from the nonlinear self-accelerating solitons. At a first analysis, one can expect that this asymmetric Airy-like pulse may lead to fairly non-trivial emission of RRs, but, to the best of our knowledge, RRs process from the nonlinear self-accelerating solitons has not been reported before.

Here we show that RRs emission from Airy pulses has a very broadband structure, due to the curved energy path of the nonlinear self-accelerating solitons. Due to the time varying velocity of these pulses, the resonant emission can be fairly more complex than for standard solitons, and the results are extremely relevant for novel broadband emission sources. Moreover, exploring these pulses in combination with RRs may improve existing application as wavelength conversion [40], biophotonics [41] and supercontinuum generation [8,25], also including quantum control [42].

2. Theoretical model and nonlinear self-accelerating pulses

We consider the generalized nonlinear Schrödinger equation (GNLSE) with higher-order dispersion [24,25]

$$i\frac{\partial u}{\partial z} + \frac{1}{2}\frac{\partial^2 u}{\partial t^2} + i\delta_3\frac{\partial^3 u}{\partial t^3} + \delta_4\frac{\partial^4 u}{\partial t^4} + \left|u\right|^2 u = 0. \tag{1}$$

We first study the effect of self-phase modulation on the Airy pulse and set $\delta_3 = \delta_4 = 0$. For this case Eq. (1) has nonlinear self-accelerating solitons with parabolic trajectory [17, 18]. Figure 1(a) shows the nonlinear self-accelerating solution with varying amplitude A (see Ref. 18). Their maximum amplitude and corresponding position first increase exponentially and then become saturated with an increasing A, as shown in Fig. 1(b). It should be pointed out that such nonlinear self-accelerating solitons are obtained numerically and have infinite energy. We use the expression of linear truncated Airy pulse as an input pulse for analytical analysis. Based on these, we fix $W = t - z^2/4$, and find a perturbative solution to Eq. (1) by considering a modified finite energy Airy pulse

$$u = NC(z)\psi(t,z) = NC(z)Ai(W + iaz)\exp\left(aW - a\frac{z^2}{4} - \frac{iz^3}{12} + \frac{ia^2z}{2} + \frac{itz}{2} \right), \quad (2)$$

with $C(z) = \exp(ik_{NL}z)$ a slowly varying term, with $|C| = 1$. The energy of the Airy pulse is $E = \int |u|^2 \, dt = N^2 e^{2a^3/3}/\sqrt{8\pi a}$ and a multiple scale approach leads to $k_{NL} = N^2 \int |Ai(t)|^4 e^{4at} dt / E(a)$. The results show the way an Airy pulse can propagate in a Kerr medium [Fig. 1(c)] by gaining a nonlinear phase shift and a corresponding nonlinear wavevector k_{NL} that depends on the Airy pulse acceleration. Figure 1(d) plots the nonlinear phase shift as a function of the parameter a and the energy E of the Airy pulse. We verified that the Airy pulse is robust with respect to self-phase modulation by numerical simulation of Eq. (1).
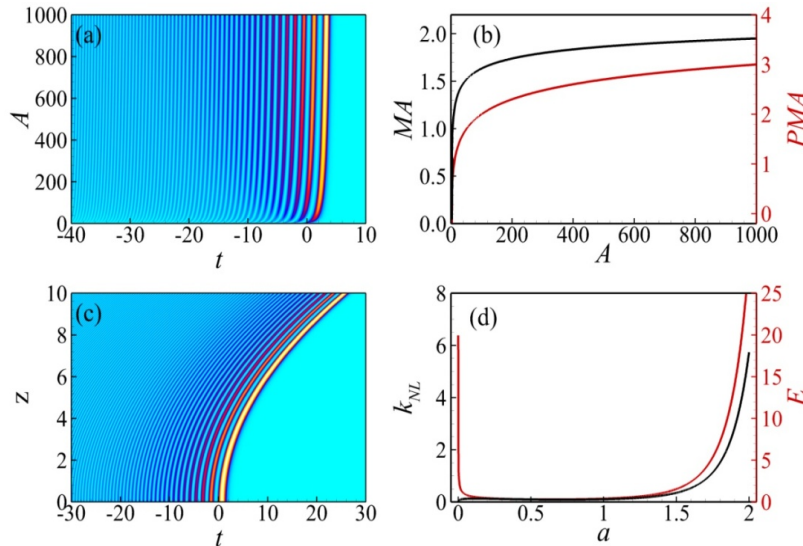

Fig. 1. (a) Amplitude of nonlinear self-accelerating solutions and (b) corresponding maximum amplitude (MA) as well as temporal positions (PMA) as a function of parameter A; (c) robust propagation of nonlinear accelerating Airy-like pulse (parameters $\delta_3 = \delta_4 = 0$), and (d) Nonlinear wave vector k_{NL} and energy E for various a.

3. Results and discussions

We then consider the effect of higher-order dispersion on the nonlinear propagation of Airy pulse. Following the approach of Akhmediev and Karlsson [24], we write the solution as $u + f$ where f is a small perturbation that satisfies the following equation

$$i\frac{\partial f}{\partial z} + \frac{1}{2}\frac{\partial^2 f}{\partial t^2} + \hat{P}(f) = -\left|u\left(t - \frac{z^2}{4}\right)\right|^2 u. \tag{3}$$

Where $\hat{P}(f) = i\delta_3\partial_t^3 f + \delta_4\partial_t^4 f$ is the higher-order dispersion contribution. A key point in our analysis is observing that the right-hand side is highly localized and moves with a curved trajectory. To derive a closed-form theory of the emission resulting from the curved path, we approximate the right-hand side of Eq. (3) to a Dirac delta with the same area by letting $\left|u\left(t - z^2/4\right)\right|^2 = E(a)\delta\left(t - z^2/4\right)$. From Eq. (3), we obtain the following result for the Fourier transform of the perturbation $\tilde{f}(\omega, L)$ for a propagation distance L,

$$\tilde{f} = iE(a)N^2\int_0^L Ai(iaz)\exp\left[-\frac{az^2}{4} + i\frac{\omega}{4}z^2 + i\frac{z^3}{24} + i\Delta k(\omega)z\right]dz, \tag{4}$$

where

$$\Delta k(\omega) = \frac{a^2}{2} + k_{NL}(a) + \frac{\omega^2}{2} - \delta_3\omega^3 - \delta_4\omega^4, \tag{5}$$

is the nonlinear phase-mismatch including the nonlinear wavevector k_{NL}. We note that $k_{NL}(a) \cong N^2/2$ for a large range of values of a.

The phase-matching condition $\Delta k(\omega_{PM}) = 0$ is the corresponding condition for resonant radiation emission at $\omega = \omega_{PM}$ for Airy pulses. This condition is similar with the case of resonant radiation generated by solitons [25].

From Eq. (4) we obtain a number of theoretical predictions, which can be written analytically by approximating $Ai(iaz) \approx Ai(0)$ because of the exponential factor in the integral that rapidly decay with respect to z. These approximations are validated below by the comparison with the numerical simulation of Eq. (1).

We first consider the case of a very long propagation $L \to \infty$ and we study the amount of generated energy at the phase-matched frequency $\Delta k = 0$ (we also neglect the quadratic and cubic term in the phase). We find

$$S_{PM} = \left|\tilde{f}(\omega_{PM}, L)\right|^2 = N^4 E(a)^2 Ai(0)^2 \left|\int_0^{L=\infty}\exp\left(-\frac{az^2}{4}\right)dz\right|^2 \tag{6}$$

$$= \frac{\pi N^4 E(a)^2 Ai(0)^2}{a}.$$

Note that because of the exponential function, the generated content does not always increase with L but saturates at a maximum value after the pump Airy pulse has spread upon evolution.

We then consider the normalized spectrum of the generated frequencies

$$s(\omega) = \frac{\tilde{f}(\omega, L = \infty)}{S_{PM}} = \int_0^\infty\exp\left[-\frac{az^2}{4} + i\frac{\omega}{4}z^2 + i\frac{z^3}{24} + i\Delta k(\omega)z\right]dz. \tag{7}$$

Neglecting the cubic term z, Eq. (7) is written in closed form as the Faddeyeva function, commonly employed in plasma physics [43],

$$s(\omega) = \sqrt{\frac{\pi}{a - i\omega}} \left[1 + iErfi\left(\frac{\Delta k}{\sqrt{a - i\omega}} \right) \right] e^{-\frac{\Delta k(\omega)^2}{a - i\omega}}. \tag{8}$$

In Fig. 2 we show the plot of the generated frequencies when varying the truncated coefficient of the Airy pulse. At strong truncation ($a = 10$) the spectrum shows distinctive peaks which correspond to the phase-matched resonant radiation as in the case of standard solitons in the case of third-order dispersion (TOD) [Fig. 2(a)] and fourth-order dispersion (FOD) [Fig. 2(b)]. When the truncated coefficient decreases ($a = 1$) satellites peak arises, which ultimately broadens in a large spectral emission mimicking the typical feature of the broadband synchrotron emission ($a = 0.1$). This behavior is found in the presence of third order dispersion, when the spectrum is asymmetrical ($\delta_3 = 0.03$, $\delta_4 = 0$), and for fourth order dispersion, when the spectrum is symmetrical as far as the resonant radiation is not dominant ($\delta_3 = 0$, $\delta_4 = 0.01$).
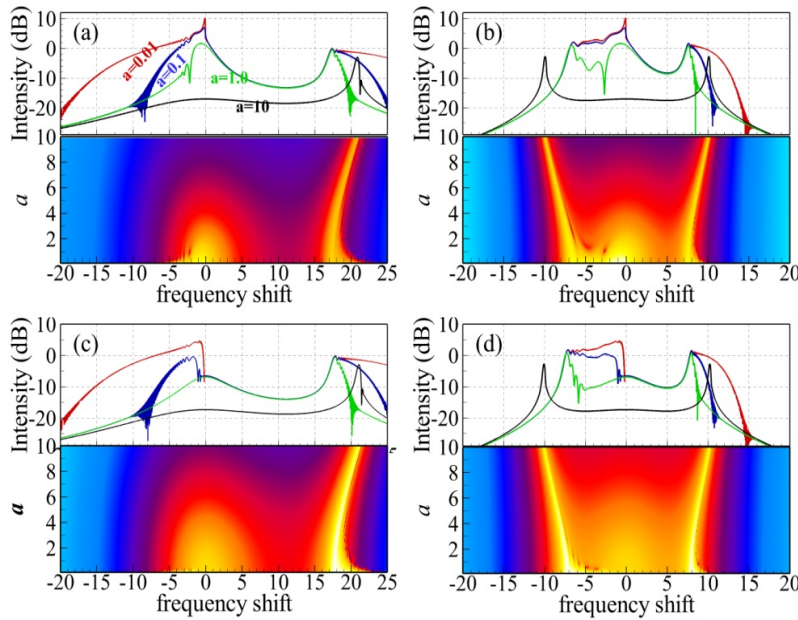

Fig. 2. Theoretically calculated emission spectrum when varying the a parameter of the Airy pulses according to $|s(\omega)|^2$ after Eq. (8) for various N, top panel $N = 1$ and bottom panel $N = 3$. For strong truncation ($a > 1$) the spectrum corresponds to resonant emission at frequencies determined by the phase-matching condition $\Delta k = 0$ after Eq. (8). For weak truncation ($a < 1$), the spectrum broadens and corresponds to the synchrotron like emission from an accelerated particle.

It should be pointed out that, in our simulations, we use a truncated nonlinear Airy pulse because nonlinear self-accelerating pulses is obtained numerically and have infinite energy. The temporal truncated positon t_c is introduced to keep $u(t < t_c) = 0$. The energy of truncated nonlinear Airy pulse is finite and increases with an increasing $|t_c|$. We test our theoretical model by solving Eq. (1) for various parameters and including TOD and FOD; as shown in Fig. 3 we find a good agreement of the generated spectrum with Eq. (8). As predicted, when the truncated coefficient of the Airy pulse decreases we observe a very broadband emission due to synchrotron emission.
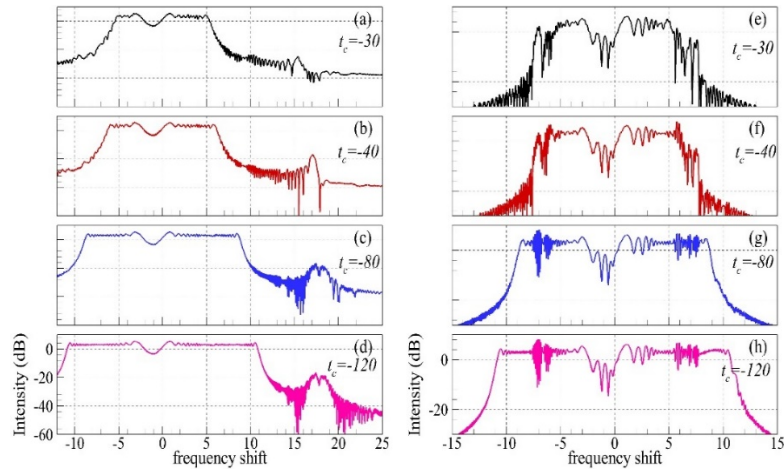
Fig. 3. Numerically calculated final spectra at $z = 2$ for nonlinear accelerating Airy-like pulse with parameter $A = 500$ and different t_c (for $u(t < t_c) = 0$) for left column $\delta_3 = 0.03$ and right column $\delta_4 = 0.01$ as obtained by Eq. (1). We obtain a very broadband emission as in Fig. 2 obtained by Eq. (1) when t_c is increased.

In our theory, it is possible to detail the dynamics of the generation of frequencies versus the propagation distance z. We adopt the stationary phase method in Eq. (7) and consider the phase term in the integral $\Phi_s(z) = z^3/24 + \omega z^2/4 + \Delta k(\omega)z$. According to the stationary phase method the leading contribution to the integral are due to the stationary points corresponding to the zeros $z_s(\omega)$ of $\Phi_s'(z) = z^2/8 + z\omega/2 + \Delta k(\omega) = 0$. For a propagation distance $z \cong 0$ in $\Phi_s'(z)$, the only dominant terms are those due to the resonant radiation $\Delta k(\omega_R) = 0$, and this regime corresponds to the standard resonant radiation as for solitons, with the difference here that the nonlinear wavevector $k_{NL}(a)$ is calculated for an Airy pulse as above. For a longer propagation, one can calculate the zeros of $\Phi_s'(z)$ which are shown in Figs. 4(a)-4(c) for some representative cases. These zeros $z_s(\omega)$ are frequency dependent; this implies that for $z > z_s(\omega)$, frequency ω is starting to be emitted. For example, Fig. 4(a) shows that at $z \cong 5$, emission in the frequency shift ranges $[-7, -5]$ and $[7, 10]$ is expected (other parameters are given in the figure). These spectral components will be progressively generated with the propagating distance. Seemingly, in Figs. 4(b) and 4(c), one can observe the spectral shift in the frequency emission. Note that the resonant frequencies such that $z_s(\omega) \cong 0$ are generated since the initial propagation. Afterwards, they shift because of the curvature in the path of the Airy pulse. Notably, only those frequencies for which $\Delta k < 0$ ($\Delta k > 0$) in order to have $z_s(\omega) > 0$ are generated, which provides a broadening that is only due to the accelerated pulse (this effect is not present in the absence of acceleration). The dynamics are clearly evident in Figs. 4(d)-4(f), where we show the calculated spectrum by the numerical solution of Eq. (1).
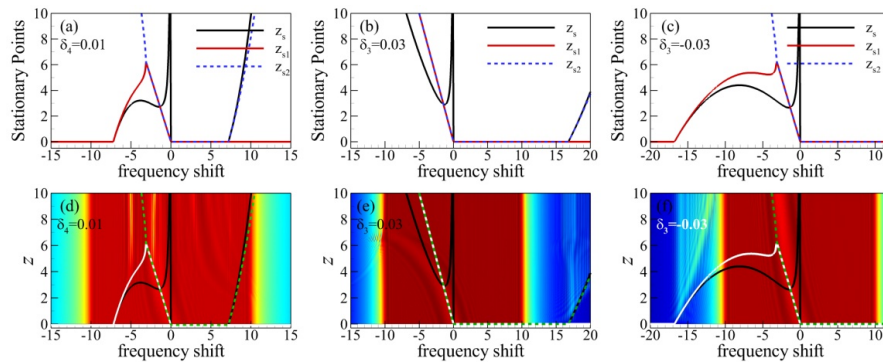
Fig. 4. (a) Calculated stationary phase point at the resonance for FOD (parameter $\delta_3 = 0.0$, $\delta_4 = 0.01$, $U(a) = 1$, $N = 1.39$) and $a = 0.01$; (d) numerical solution of the NLSE showing the synchrotron-like emission; as in (b, e) and (c, f) with positive and negative TOD ($\delta_3 = \pm 0.03$) respectively.

4. Conclusion

In conclusion, we have theoretically predicted and numerically verified that the curved trajectory of an Airy beam in the time domain allows for generating very broadband emission. This effect strongly resembles synchrotron radiation, which is the broadband emission of electromagnetic radiation by charged particles accelerated on curved trajectories. In simple terms, the spectral content can be explained by resonant radiation emission, because the solitonic dynamics of the Airy pulse emit resonant frequencies, as it happens for standard solitons propagating at a constant speed. However, as the velocity of the Airy pulse changes with time, also the emitted frequencies change, resulting in a very broad band emission. We reported an analytic theory for the generated spectrum and its dynamics.

As the curved trajectory of the pulses can be optimized by a proper design of the fiber parameters, eventually varying with respect to propagation distance by dispersion management, one can produce oscillating pulses with periodic velocities or time dependent accelerations. The generated spectrum will depend on the specific trajectory and can be predicted by the theory here reported. We expect that this specific approach may lead to novel broadband sources and supercontinuum emission with engineered spectral for applications in spectroscopy, metrology and quantum sources.