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Effects of correlations between particle longitudinal positions and transverse plane on bunch length measurement: a case study on GBS electron LINAC at ELI-NP

L Sabato[1], P Arpaia[2], A Cianchi[3], A Liccardo[2], A Mostacci[4], L Palumbo[4] and A Variola[5]

[1] Department of Engineering, University of Sannio, Corso G. Garibaldi, 107, Benevento, Italy
[2] Department of Electrical Engineering and Information Technology, University of Naples Federico II, Via Claudio 21, Naples, Italy
[3] University of Rome Tor Vergata, Via della Ricerca Scientifica 1, Rome, Italy
[4] Department Scienze di Base e Applicate per l'Ingegneria (SBAI), University of Rome Sapienza, Via Antonio Scarpa, 14, Rome, Italy
[5] Istituto Nazionale di Fisica Nucleare—Laboratori Nazionali di Frascati (INFN–LNF), Via Enrico Fermi, 40, Frascati, Italy

E-mail: luca.sabato@unisannio.it

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Abstract

In high-brightness LINear ACcelerators (LINACs), electron bunch length can be measured indirectly by a radio frequency deflector (RFD). In this paper, the accuracy loss arising from non-negligible correlations between particle longitudinal positions and the transverse plane (in particular the vertical one) at RFD entrance is analytically assessed. Theoretical predictions are compared with simulation results, obtained by means of ELEctron Generation ANd Tracking (ELEGANT) code, in the case study of the gamma beam system (GBS) at the extreme light infrastructure—nuclear physics (ELI-NP). In particular, the relative error affecting the bunch length measurement, for bunches characterized by both energy chirp and fixed correlation coefficients between longitudinal particle positions and the vertical plane, is reported. Moreover, the relative error versus the correlation coefficients is shown for fixed RFD phase 0 rad and π rad. The relationship between relative error and correlations factors can help the decision of using the bunch length measurement technique with one or two vertical spot size measurements in order to cancel the correlations contribution. In the case of the GBS electron LINAC, the misalignment of one of the quadrupoles before the RFD between -2 mm and 2 mm leads to a relative error less than 5%. The misalignment of the first C-band accelerating section between -2 mm and 2 mm could lead to a relative error up to 10%.

Keywords: electron bunch length, RF deflector, ELI-NP, gamma beam system, LINAC, correlations between particle longitudinal positions and vertical plane

(Some figures may appear in colour only in the online journal)

1. Introduction

Electron bunch length in high-brightness LINear ACcelerators (LINACs) can be measured indirectly by means of a radio frequency deflector (RFD) or a transverse deflecting structure (TDS) [1, 2]. An RFD provides a transverse kick to the electron bunch introducing a relationship between its dimensions, longitudinal (i.e. the length) and transverse, on a screen, placed in front of the RFD. In this paper, RFDs providing vertical kicks are considered without losing generality. Therefore, the bunch length can be assessed through vertical spot size measurements on the screen, and through the RFD characteristics [3, 4]. These characteristics are obtained by an appropriate calibration for measuring the deviation of the vertical bunch centroid associated with the deflecting voltage phase [5]. This measurement method is widely used around the world: at Stanford Linear Accelerator Center (SLAC) [6], at Deutsches Elektronen–Synchrotron (DESY) [7], at Massachusetts Institute of Technology (MIT) Plasma Science and Fusion Center (PSFC) [8], at sources for plasma accelerators and radiation Compton with lasers and beams (SPARC-LAB) [9], and so on.

The assumptions about the bunch proprieties at RFD entrance could be sources of systematic errors in length measurements. In particular, when at RFD entrance the bunch has a non-negligible correlation between particle longitudinal positions and energies (i.e. an *energy chirp*), the length measurement is affected by a deterministic intrinsic error, directly related to the RFD phase offset [10]. Furthermore, the bunch could be characterized by non-negligible correlations between particle longitudinal positions and the vertical plane at RFD entrance and, therefore, the measurement accuracy could be deteriorated. The correlations between particle longitudinal positions and vertical plane are introduced by transverse wake-field of the accelerating sections upstream of the RFD. This happens especially in presence of off-axis kick, RFD vertical deflections (the vertical kick depends on the particle longitudinal position) [11], and quadrupole misalignments. The RFD introduces these correlations intrinsically, owing its working principle, but the correlations due to misalignments of accelerating sections and/or quadrupoles upstream of the RFD are undesired effects, capable of affecting significantly the bunch length measurements.

The case study is the gamma beam system (GBS) electron LINAC at the extreme light infrastructure-nuclear physics (ELI-NP) [12]. The GBS at ELI-NP will be an advanced source of up to 20 MeV gamma rays based on Compton backscattering [13]. The specifications on the requested spectral density of the gamma ray (10^4 photon/(eV·s)) cannot be achieved with single bunch collisions at room temperature [14], because The GBS electron LINAC can run at a maximum repetition rate of 100 Hz [15], and therefore the final optimization foresees multiple bunch collisions. The electron bunch length must be tuned according to the laser bunch length. Consequently, the measurement of the properties of the single bunch and the whole train of bunches is a crucial task for beam diagnostic [16].

The effects of the correlations between particle longitudinal positions and the vertical plane on the vertical spot size at the screen are evaluated in [17]. The variation of the vertical spot size at screen due to these correlations produces a relative error on the bunch length measurement, because the vertical spot size changes without a changing of the measurand. These effects can be cancelled by measuring the vertical spot size with two zero-crossing phases [18]. This procedure has been widely used in RFD diagnostics, but there are cases where two phases are not used, because based on a single-shot measurement intrinsically, or requiring only a one zero-crossing phase procedure, for rapidity and simplicity. This measurement technique is affected by a systematic error if correlations between particle longitudinal positions and vertical plane are significant.

In this paper, the accuracy loss of bunch length measurements due to the correlations among longitudinal particle positions and their vertical positions and divergences is assessed analytically. Then, theoretical predictions are compared with simulation results obtained by ELEctron Generation ANd Tracking (ELEGANT) code. In particular, after introducing the measurement technique in sections 2, in 3, the effects of the correlations between particle longitudinal positions and vertical plane at RFD entrance are discussed. Simulations results of a case study on the gamma beam system (GBS) electron LINAC at the extreme light infrastructure-nuclear physics (ELI-NP) [12] are presented in section 4. In particular, the relative error affecting the bunch length measurement, in the case of bunches characterized by the energy chirp and otherwise, with fixed correlation coefficients between longitudinal particle positions and the vertical plane, is reported. Moreover, the relative error versus the correlation coefficients is shown for fixed RFD phase of 0 rad and π rad. The chosen RFD gives a vertical kick to the bunch, and hence is a vertical deflector.

2. Background on bunch length measurement

In this section, the working principle of the bunch length measurement is reviewed. In particular, the general equation of the vertical spot size at screen is shown in section 2.1. Then, the analytical model of the one zero-crossing phase measurement technique, not considering correlation between vertical and longitudinal planes, are described in section 2.2.

2.1. Working principle

The measurement method is based on the idea that the vertical spot size at the screen, placed after an RFD, changes when the RFD is turned on, because the RFD voltage introduces a variation of the bunch vertical dimension, depending on the bunch longitudinal dimension [9]. In particular, the deflecting voltage phase is chosen in order to have a zero crossing of the transverse voltage in the centre of the bunch (0 rad or π rad), and to give a linear transverse deflection from head to tail. In this way, the spread out of the bunch in the vertical axis and, therefore, the measurement resolution, are maximized.

The bunch length, thus, can be assessed from vertical spot size measurements at screen, with RFD on and off, after an appropriate calibration of the variation of the position of vertical bunch centroid associated with the deflecting voltage phase [5].

2.2. Theory

Assuming that: (i) there is only a drift space between the RFD and the screen, (ii) the RFD is modelled as the cascade of a drift of length $L_{\mathrm{RFD}}/2$, a vertical kicker, and another $L_{\mathrm{RFD}}/2$-long drift, where L_{RFD} is the mechanical length of the RFD [10], (iii) the vertical bunch centroid at RFD center C_{y_0}, the longitudinal bunch centroid at RFD center C_{z_0} and the average of the particle vertical divergences $C_{y_0'}$ are null, and (iv) all particles have the same energy, the vertical spot size is:

$$\sigma_{y_s}^2(\varphi) = \sigma_{y_s,\mathrm{off}}^2 + K_{\mathrm{cal}}^2(\varphi)\sigma_{t_0}^2 + 2K_{\mathrm{cal}}(\varphi)\left[\sigma_{y_0t_0} + L\sigma_{y_0't_0}\right], \tag{1}$$

where φ is the deflecting voltage phase, $\sigma_{y_s,\mathrm{off}}$ the vertical spot size with RFD off [19], σ_{t_0} the bunch length in seconds, $\sigma_{y_0t_0}$ and $\sigma_{y_0't_0}$ the covariances between particle longitudinal and vertical positions and between particle longitudinal positions and vertical divergences at RFD centre, respectively, and K_{cal} is the calibration factor:

$$K_{\mathrm{cal}}(\varphi) = 2\pi f_{\mathrm{RF}} L C_{\mathrm{rfd},a}\cos(\varphi), \tag{2}$$

where f_{RF} is the frequency of the deflecting voltage, L the distance between RFD centre and the screen, and $C_{\mathrm{rfd},a}$ the ratio $V_t/\langle E\rangle$ in the ultra-relativistic regime, where V_t is the deflecting voltage amplitude and $\langle E\rangle$ the particles energy average in MeV.

2.3. One zero-crossing phase measurement technique

In the one zero-crossing phase bunch length measurement, only the first two terms of (1) are considered [6, 9, 20]:

$$\sigma_{y_s}^2 = \sigma_{y_s,\mathrm{off}}^2 + K_{\mathrm{cal}}^2(\varphi)\sigma_{t_0}^2. \tag{3}$$

The first term is the vertical spot size at screen when the RFD is off, taking into account the bunch vertical parameters at screen [19]. In order to achieve a better resolution, $\sigma_{y_s,\mathrm{off}}$ has to be designed as smaller as possible by means of vertical focusing quadrupole, placed before the RFD. The second term of (1) contains the information about the measurand (i.e. the bunch length σ_{t_0}). In the second term, the calibration factor appears, calculated from vertical bunch centroid measurements at screen for different RFD phases [5]:

$$K_{\mathrm{cal}} = 2\pi f_{\mathrm{RF}}\frac{\mathrm{d}C_{y_s}}{\mathrm{d}\varphi}, \tag{4}$$

where the vertical bunch centroid at screen C_{y_s} is [6, 20]:

$$C_{y_s} = LC_{\mathrm{rfd},a}\sin(\varphi). \tag{5}$$

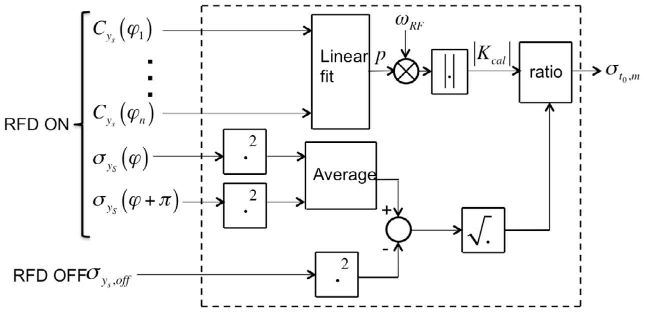


Figure 1. Proposed model of the measurement production of the bunch length.

The resolution can be improved (i) by increasing the calibration factor (i.e. the deflecting voltage amplitude and the distance between the RFD and the screen, assuming the vertical spot size at screen with RFD off can be kept constant), and (ii) by decreasing $\sigma_{y_s,\mathrm{off}}$, by means of vertical focusing quadrupoles placed before RFD [10]. The bunch length measurement can be obtained from (3):

$$\sigma_{t_0,m} = \frac{\sqrt{\sigma_{y_s}^2 - \sigma_{y_s,\mathrm{off}}^2}}{|K_{\mathrm{cal}}|}. \tag{6}$$

The model of measurement production is detailed in [10].

3. Effects of correlations between particle longitudinal positions and vertical plane

In this section, firstly, the additional steps of the model of measurement production in order to cancel the contributions of the covariances $\sigma_{y_0t_0}$ and $\sigma_{y_0't_0}$ are discussed in section 3.1. Then, the analytical assessment of the relative error of the bunch length measurement introduced by these correlations is treated in section 3.2. Finally, the measurement relative error in the case of a bunch with also a non-negligible energy chirp at RFD entrance is determined (section 3.3).

3.1. Proposed model of measurement production

With respect to the proposal in [10], the covariances $\sigma_{y_0t_0}$ and $\sigma_{y_0't_0}$ add the third term of (1) in the vertical spot size at screen with RFD on. The effects of this term on the bunch length measurements are mitigated with respect to the one zero-crossing phase technique by an additional vertical spot size measurement. By measuring the vertical spot size with the phase of the deflection voltage equal to φ and $\varphi+\pi$, the unique term of the (1) changing in sign but not in value is the third term, i.e. the contribution of the covariances $\sigma_{y_0t_0}$ and $\sigma_{y_0't_0}$. Conversely, the other two terms keep the same sign and value. This consideration suggests a simple method to compensate the covariances effect, namely averaging $\sigma_{y_s}^2(\varphi)$ and $\sigma_{y_s}^2(\varphi+\pi)$ [17]:

$$\overline{\sigma_{y_s}^2}(\varphi) = \frac{\sigma_{y_s}^2(\varphi) + \sigma_{y_s}^2(\varphi+\pi)}{2} = \sigma_{y_s,\mathrm{off}}^2 + K_{\mathrm{cal}}^2(\varphi)\sigma_{t_0}^2. \tag{7}$$

An improved model of measurement production of the bunch length can be developed (figure 1). The model relies on two main stages: the measurement and the data processing. The former stage consists of the following operations:

Table 1. GBS electron LINAC bunch parameters of vertical plane at RFD entrance. σ_y and $\sigma_{y'}$ are expressed in rms.

C_y (nm)	$C_{y'}$ (nrad)	σ_y (mm)	$\sigma_{y'}$ (μm)	$\sigma_{yy'}$ (mm·mrad)
0.655	-3.36	0.354	57.6	$-2.03 \cdot 10^{-2}$

Table 2. GBS electron LINAC bunch parameters of longitudinal plane at RFD entrance, where $\delta = (E - \langle E \rangle)/\langle E \rangle$ and $r_{t\delta} = \sigma_{t\delta}/(\sigma_{t_0}\sigma_\delta)$. σ_{t_0} and σ_δ are expressed in rms.

$\langle E \rangle$ (MeV)	σ_{t_0} (ps)	σ_δ	$r_{t\delta}$
118	0.912	$6.05 \cdot 10^{-3}$	0.965

(i) RFD off: measurement of the vertical spot size at screen $\sigma_{y_s,\mathrm{off}}$;
(ii) RFD on: measurement of the vertical bunch centroid at screen for different values of RFD phase φ_i (centred in 0 rad or π rad) $C_{y_s}(\varphi_i)$;
(iii) RFD on: measurement of the vertical spot size at screen with RFD phase 0 rad $\sigma_{y_s}(0)$;
(iv) RFD on: measurement of the vertical spot size at screen with RFD phase π rad $\sigma_{y_s}(\pi)$.

The data processing stage consists of the following steps:

(i) estimation of the slope p of the plot vertical bunch centroid at screen versus φ by means of a linear fit, and then calculate the calibration factor K_{cal} by multiplying the deflecting voltage angular frequency by the slope p (4);
(ii) estimation of the average $\overline{\sigma_{y_s}^2}$ between the squared values of the vertical spot sizes $\sigma_{y_s}(0)$ and $\sigma_{y_s}(\pi)$ (from (7));
(iii) evaluation of the bunch length $\sigma_{t_0,m}$ (from (7)):

$$\overline{\sigma}_{t_0,m} = \frac{\sqrt{\overline{\sigma_{y_s}^2} - \sigma_{y_s,\mathrm{off}}^2}}{|K_{\mathrm{cal}}|}. \tag{8}$$

The first and the second step of the measurement stage are the same as the one zero-crossing phase measurement production in [10]. The third and the fourth steps consist of two measurements of the vertical spot size at screen $\sigma_{y_s}(\varphi)$ with RFD phase 0 rad and π rad, respectively. The first step of the data processing (i.e. the assessment of the calibration factor), also, is the same as the one zero-crossing phase. In the second step, $\overline{\sigma_{y_s}^2}$ and the squared values of the vertical spot sizes $\sigma_{y_s}(0)$ and $\sigma_{y_s}(\pi)$ are averaged. Finally, the third step assesses the bunch length by means of (8).

3.2. Analytical assessment

The covariances $\sigma_{y_0t_0}$ and $\sigma_{y'_0t_0}$ do not affect the vertical bunch centroid at screen (5), and, therefore, the calibration factor. However, they add a contribution to the vertical spot size at the screen (i.e. the third term of (1)), and thus, they affect the measurement. The correlations between particle longitudinal positions and vertical plane is introduced by transverse wakefield of the accelerating sections upstream of the RFD. This happens especially in presence of off-axis kick, RFD vertical deflections (the vertical kick depends on the particle longitudinal position) [11], and quadrupole misalignments, because a vertical misalignment gives a vertical kick to the particles of the bunch, changing its vertical direction ($\langle y'_0 \rangle \neq 0$) and, therefore, the following quadrupoles give vertical kicks according to the particle longitudinal position [21]. The RFD introduces these correlations intrinsically, namely owing to its working principle. However, the correlations due to misalignments of accelerating sections and/or quadrupoles upstream of the RFD are undesired effects and affect the measurements accuracy. In particular, the absolute value of the third term of (1) is maximum when the working RFD phase is 0 rad or π rad. This contribution gives rise to a measurement relative error, according to (6):

$$E_r = \frac{|\sigma_{t_0} - \sigma_{t_0,m}|}{\sigma_{t_0}}, \tag{9}$$

where the actual measured bunch length $\sigma_{t_0,m}$ is (substituting (1) in (6)):

$$\sigma_{t_0,m}(\varphi) = \sqrt{\sigma_{t_0}^2 + \frac{2}{K_{\mathrm{cal}}(\varphi)}\left[\sigma_{y_0t_0} + L\sigma_{y'_0t_0}\right]}. \tag{10}$$

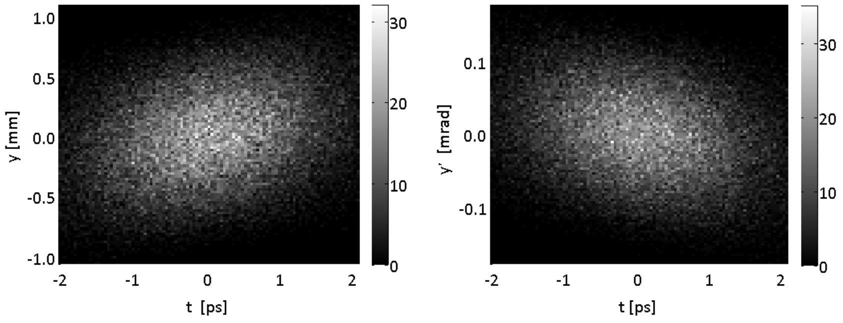
The measurement relative error is minimum when the RFD phase is 0 rad or π rad (see (9) and (10)). The actual measured bunch length can be expressed in function of the correlation coefficients between particle longitudinal positions and vertical positions $r_{y_0t_0} = \sigma_{y_0t_0}/(\sigma_{y_0}\sigma_{t_0})$ and between particle longitudinal positions and vertical divergences $r_{y'_0t_0} = \sigma_{y'_0t_0}/(\sigma_{y'_0}\sigma_{t_0})$ at RFD centre for a fixed RFD phase:

$$\sigma_{t_0,m}\left(r_{y_0t_0}, r_{y'_0t_0}\right) = \sigma_{t_0}\sqrt{1 + \frac{2}{\sigma_{t_0}K_{\mathrm{cal}}}\left[r_{y_0t_0}\sigma_{y_0} + Lr_{y'_0t_0}\sigma_{y'_0}\right]}. \tag{11}$$

The relative error is minimum for small values of the correlation coefficient, and it can be huge according to r_{yt} and $r_{y't}$. In particular, the relative error is minimum when second term of (11) is null and, therefore, for the values (r_{yt},$r_{y't}$) lying on the straight line defined by the equation:

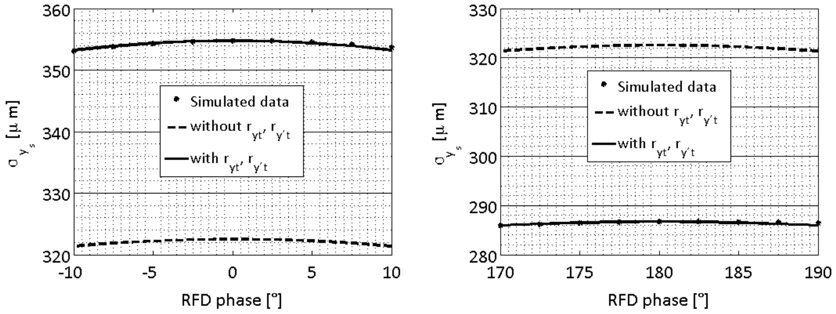
$$r_{y't} = -\frac{\sigma_{y_0}}{L\sigma_{y'_0}}r_{yt}. \tag{12}$$

The straight line has a negative slope and passes through the origin. Moreover, the contribution to the vertical spot size with RFD on of these correlations (i.e. the third term of the (1)) can be negative and greater than the second term for some values of (r_{yt},$r_{y't}$). In this case, the vertical spot size with RFD on becomes smaller than the vertical spot size with RFD off. In fact, the RFD does not spread out the bunch in the vertical plane, but focuses it in the vertical plane. For these values of (r_{yt},$r_{y't}$), the one zero-crossing phase measurement technique in [10] gives an imaginary value for the bunch length (from (6)). The function of the relative error versus the correlation coefficients (11) can help the decision about using the bunch length measurement technique with one or two vertical spot size measurements in order to cancel the correlation contributions. The decision depends on the estimated values of these correlations and if the consequent relative error can be accepted.



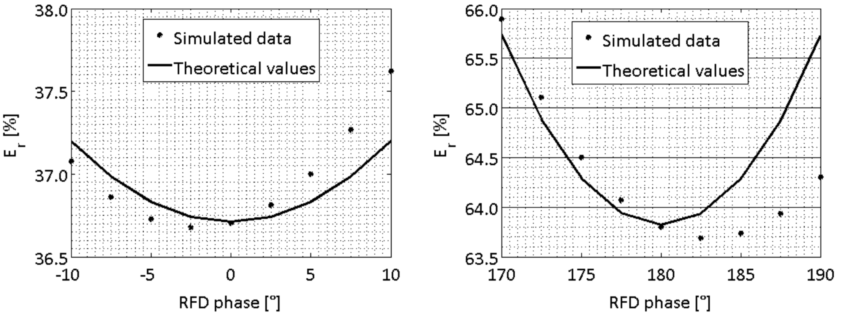
(a) (b)

Figure 2. (a) Particle vertical and longitudinal positions trace space of the bunch at RFD entrance. (b) Particle vertical divergences and longitudinal positions trace space of the bunch at RFD entrance.



(a) (b)

Figure 3. Vertical spot size at screen around the RFD phase of (a) 0 rad and (b) π rad. Simulated data in stars, theoretical values without the correlation contribution (3) in dashed line, and theoretical values with the correlation contribution (1) in solid line.



(a) (b)

Figure 4. Bunch length measurement relative error around the RFD phase of (a) 0 rad and (b) π rad. Simulated data in stars, theoretical values (14) in solid line.

3.3. Energy chirp contribution

The contribution of the energy chirp of the bunch at RFD entrance on the vertical spot size at screen is explained in [10]. It can alter the vertical spot size after the RFD according to the last term of the following equation [10]:

$$\sigma_{y_s}^2(\varphi) = \sigma_{y_s,\mathrm{off}}^2 + K_{\mathrm{cal}}^2(\varphi)\sigma_{t_0}^2 + 2K_{\mathrm{cal}}(\varphi)\left[\sigma_{y_0t_0} + L\sigma_{y_0't_0}\right] - L^2\omega_{\mathrm{RF}}C_{\mathrm{rfd},a}^2\sin\left(2\varphi\right)\sigma_{t_0\delta}. \tag{13}$$

The last term of (13) is the energy chirp contribution and is minimum when the RFD phase is 0 rad (or π rad). It cannot be cancelled using the improved measurement procedure in section 3. The measured bunch length using the measurement procedure in section 2 for a bunch with a non-negligible energy chirp is:

$$\sigma_{t_0,m} = \sqrt{\sigma_{t_0}^2 + \frac{2}{K_{\mathrm{cal}}}\left[\sigma_{y_0t_0} + L\sigma_{y_0't_0}\right] - \frac{L^2\omega_{\mathrm{RF}}C_{\mathrm{rfd},a}^2\sin\left(2\varphi\right)}{K_{\mathrm{cal}}^2}\sigma_{t_0\delta}}. \tag{14}$$

4. Case study on GBS electron Linac at ELI-NP

In this section, the above accuracy loss assessment is validated by referring to a case study on the GBS electron LINAC at the ELI-NP [12]. The simulation results, obtained by means of ELEGANT code [22], are compared with the theoretical predictions. Then, the simulations conditions are introduced (section 4.2), and then the numerical results are presented (section 4.3).

4.1. Case study environment

ELI-NP is one of the pillars of the ELI European project dedicated to high-level research on ultra-high intensity laser, laser-matter interaction, and secondary sources [23]. The GBS at ELI-NP will be built in Magurele, Romania, and it will be an advanced source of up to 20 MeV gamma rays based on Compton back-scattering, i.e. collision of an intense high power laser beam and a high brightness electron beam with maximum kinetic energy of about 720 MeV [14]. The purposes of this machine are various: coverage of frontier fundamental physics, new nuclear physics and astrophysics research topics as well as applications in material and life sciences, industrial tomography, nuclear waste management [23]. High performance diagnostic is mandatory in order to achieve high brightness in high repetition rate machine. The GBS electron LINAC can run at a maximum repetition rate of 100 Hz [15], and therefore the specifications on the requested spectral density (10^4 photon/(eV·s)) cannot be achieved with single bunch collisions at room temperature [14]. The final optimization foresees multiple bunch collisions, with trains of 32 electron bunches, 1 ps in length, with a repetition period of 16 ns, distributed along a 500 ns RF pulse. The laser pulse should collide with all the electron bunches in the RF pulse before being dumped, therefore the RF LINAC has to provide bunches in each train equally spaced for recirculating the laser pulse in a suitable device [14]. Moreover, the electron bunch length must be tuned according to the laser bunch length. Consequently, the measurement of the properties of the single bunch and the whole train of bunches is a crucial task for beam diagnostic [16, 24, 25].

4.2. Simulation conditions

The RFD is modelled as a travelling wave cavity in ELEGANT (RFDF element). This cavity provides a constant transverse deflection along the transverse coordinates. It is probably the best actual model, because real cavities contain a mixture of TM- and TE-like modes resulting in a uniform deflection [26]. The parameters of the RFDF element are: frequency 2.856 GHz, length 0.256 m, amplitude voltage 1 MV, tilt angle $\pi/2$ rad in order to have a vertical deflection, phase $-\pi/2$ rad (the phase in the RFDF element $\Delta\phi$ is different from the presented theoretical model φ, because the deflecting voltage in our theoretical model is defined as: $V(z_0) = V_t \sin(2\pi f_{\mathrm{RF}} z_0/c + \varphi)$; on the contrary the deflecting voltage in RFDF element $V(z_0) = V_t \cos(2\pi f_{\mathrm{RF}} z_0/c + \Delta\phi)$; therefore, the relationship between the phases is: $\Delta\phi = \varphi + \pi/2$). Downstream of the RFD is placed a drift (DRIF element, length 1.01 m) and a screen (a watching point in ELEGANT).

A bunch composed of 50 000 particles with a charge of 250 pC was tracked by means of ELEGANT code from the RFD to the screen of GBS electron LINAC, placed between the first and the second C-band accelerating section of GBS electron LINAC [14]. The deflecting voltage amplitude V_t and frequency are 1 MV and 2.856 GHz, respectively. The distance between RFD center and the screen L is 1.1380 m. The GBS electron LINAC bunch parameters of vertical and longitudinal planes at RFD entrance, are reported in tables 1 and 2, respectively. The bunch is traveling in parallel to the accelerator axis ($C_{y'} \approx 0$ rad) and just on the axis itself ($C_y \approx 0$ rad). The vertical spot size at screen with RFD off is 281 μm.

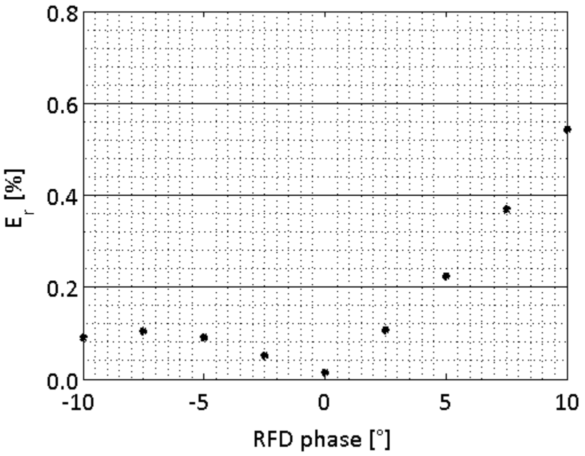
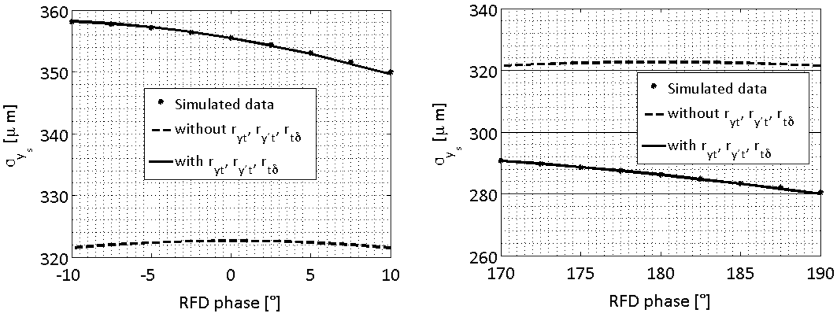


Figure 5. Bunch length measurement error using the proposed model of measurement production (8).

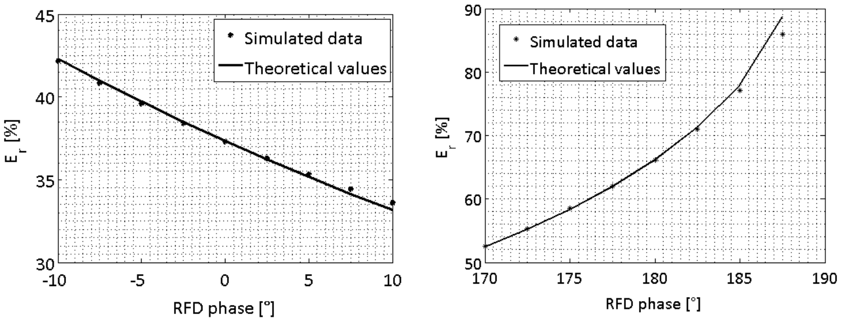
4.3. Numerical results

The bunch is chosen with non-negligible correlations between particle vertical and longitudinal positions and between vertical divergences and longitudinal positions at RFD entrance. In particular, the correlation coefficients are chosen: $r_{yt} = 0.25$ and $r_{y't} = -0.25$, as shown by the longitudinal positions and vertical plane trace spaces of the bunch at RFD entrance in figure 2. The chosen correlation coefficients are large (even though the correlation coefficients $r_{ij} < |1|$): in the case of the GBS electron LINAC assuming the absence of correlations between particle longitudinal positions and vertical plane before the first accelerating section, for the misalignments of the first accelerating section and the first quadrupole upstream of the RFD of -2 mm, the correlation coefficients at RFD entrance are (i) $r_{yt} = 0.107$ and (ii) $r_{y't} = 0.290$ [21]. Moreover, the numerical results are presented in both the cases of a bunch characterized with negligible ($r_{t\delta} = -1.03 \cdot 10^{-10}$) and non-negligible ($r_{t\delta} = 0.965$) energy chirp. Firstly, the prediction goodness of the vertical spot size at screen by means of the (3) used in the one zero-crossing phase model of measurement production is analyzed. Then, the relative error is assessed: $E_r = |\sigma_{t_0} - \sigma_{t_0,m}|/\sigma_{t_0}$, where σ_{t_0} is the true bunch length and $\sigma_{t_0,m}$ is the measured bunch length obtained by means of the one zero-crossing phase model of measurement production [10]. The theoretical calibration factor is calculated from equation (2), using the values of f_{RF}, L, V_t, and $\langle E\rangle$ reported in section 4.2 and table 2. The calibration factor, calculated from vertical bunch centroid measurements (3 measurements in 87.27 mrad), has a negligible error less than 0.08% [19]. Finally, the relative error of the bunch length



(a) (b)

Figure 6. Vertical spot size at screen around the RFD phase of (a) 0 rad and (b) π rad: simulated data (stars), theoretical values without (dashed line, (3)) and with (solid line, (13)) correlation and chirp contributions.



(a) (b)

Figure 7. Bunch length measurement relative error around the RFD phase of (a) 0 rad and (b) π rad. Simulated data in stars, theoretical values (14) in solid line.

measurement versus the correlation coefficients (between -1 and 1, every possible values) is shown.

4.3.1. Bunch characterized with a negligible energy chirp. In figure 3, theoretical predictions (3) and (1) and simulated data of the vertical spot size at screen are compared at varying RFD phase centred in (a) 0 rad and (b) π rad (the working RFD phases). The comparison shows a satisfying agreement. Conversely, the theoretical predictions given by (3) can reach a relative error of 12% for $\varphi = \pi$ with respect to the simulated data.

In figure 4, the relative error of the bunch length measurement is plotted versus the RFD phase, for both simulated data (stars) and theoretical predictions from (9) (solid line), for (a) 0 rad and (b) π rad. The theoretical relative error explains only the contribution of the correlations between particle longitudinal positions and the vertical plane using the one zero-crossing phase model of the measurement production [10]. The bunch length measurement relative error is not negligible (see figure 4): the relative error is minimum in 0 rad (about 37%) and π rad (about 64%).

By the model of the measurement production proposed in section 3, the bunch length relative error is reduced significantly (see figure 5): within an RFD phase offset of $\pi/18$ rad relative error is smaller than 0.6%.

4.3.2. Bunch characterized with a non-negligible energy chirp. In the case of GBS electron LINAC, the bunch at the RFD entrance is characterized by a non-negligible energy chirp [10]. In the following simulations, the values of RFD and bunch parameters are reported in section, as well as in tables 1 and 2. The values are the same of the previous case study excepting a non-negligible energy chirp $r_{t\delta} = 0.965$. In figure 6, the effects of the energy chirp on vertical spot size at screen can be noticed. The contribution of the energy chirp on the vertical spot size at screen is not symmetric around the RFD phase of 0 rad (or π rad). Conversely, the contribution of the correlations between the particle longitudinal positions and the vertical plane is symmetric around these two RFD

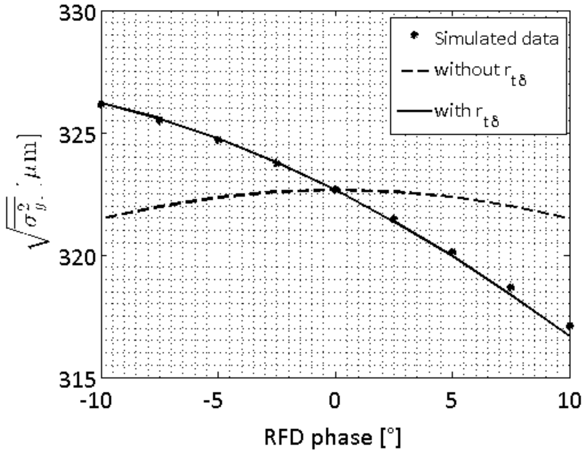
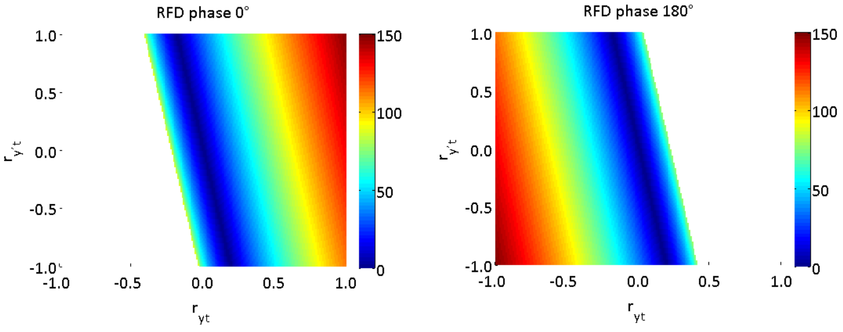


Figure 8. Square root of $\overline{\sigma^2_{y_s}}$ versus RFD phase. Simulated data in stars, theoretical values without the energy chirp contribution (3) in dashed line, and theoretical values with the chirp contribution (13) in solid line.



(a) (b)

Figure 9. Relative length error, when the model of section 2 for a bunch with non-negligible correlations between particle longitudinal positions and the vertical plane at RFD entrance (9), versus correlation coefficients r_{yt} and $r_{y't}$ with RFD phase (a) 0 rad and (b) π rad.

phases (figure 3). The vertical spot size at screen with RFD on can be smaller than with RFD off (281 μm [19]), i.e. the bunch is slightly focused vertically by the RFD (for $\varphi = \pi$). Applying the measurement procedure of section 2, the bunch length measurement relative error versus RFD phase is depicted in figure 7. The theoretical relative error explains the contribution of the correlations between particle longitudinal positions and the vertical plane and of the energy chirp using the model of the measurement production of section 2. The bunch length measurement error is about 37% and 66% for an RFD phase of 0 rad and π rad, respectively.

The square root of $\overline{\sigma_{y_s}^2}$ is reported in figure 8. The contribution on $\overline{\sigma_{y_s}^2}$ of the correlations between particle longitudinal positions and the vertical plane is now canceled. The energy chirp contribution, conversely, continues to affect the vertical spot size at screen. The term $\sqrt{\overline{\sigma_{y_s}^2}}$ is the same vertical spot size at the screen as an electron bunch with negligible correlations between particle longitudinal positions and vertical plane and a non-negligible energy chirp at RFD entrance [10]. For these reasons, applying the proposed model of measurement production in section 3, the bunch length measurement relative error is the same as in case of a bunch without correlations [10].

4.3.3. Bunch length relative error at varying correlation coefficients. When the one zero-crossing phase model in [10] is used for a bunch with non-negligible correlations between particle longitudinal positions and the vertical plane at RFD entrance, the measurement is affected by a systematic error (10). For a fixed RFD phase 0 rad (or π rad), this error depends on the correlation coefficients r_{yt} and $r_{y't}$, as reported in figure 9. The relative error is minimum for small values of the correlation coefficients, and it can be huge according to r_{yt} and $r_{y't}$. The white areas in figure 9 are the values of (r_{yt},$r_{y't}$) for which the RFD focuses the bunch in the vertical plane and, therefore, the one zero-crossing phase measurement technique does not provide a real value for the bunch length (from (6)).

In the case of the GBS electron LINAC, the misalignment of one of the quadrupoles, before the RFD, between -2 mm and 2 mm leads to a relative error less than 5%. The misalignment of the first C-band accelerating section between -2 mm and 2 mm could lead to a relative error up to 10%.

5. Conclusions

The correlations between particle longitudinal positions and the vertical plane at RFD entrance could affect the accuracy of bunch length measurements. These correlations do not affect the vertical bunch centroid at screen and, therefore, the calibration factor. Conversely, they add a contribution to the vertical spot size at the screen with RFD on and, therefore, introduce a systematic error on bunch length measurement. The systematic error due to these correlations is analytically assessed and compared with simulation results obtained by means of ELEGANT code for tracking the particles from RFD to the screen in the GBS electron LINAC case at ELI-NP. In particular, the vertical spot size at screen and the relative error of the bunch length measurement for a bunch both with and without energy chirp at RFD entrance and fixed correlation coefficients between longitudinal particle positions and the vertical plane are compared. Finally, the relative error versus the correlation coefficients is shown for fixed RFD phase 0 rad and π rad. The relative error is minimum for small values of the correlation coefficients, and it can be huge for big values of r_{yt} and $r_{y't}$. The observation of the plot of the relative error versus correlations factors can help the decision of using the bunch length measurement technique with one or two vertical spot size measurements in order to cancel the contribution of the correlations. The decision depends on the estimation of the values of these correlations and if the introduced relative error can be accepted. In the case of the GBS electron LINAC, the misalignment of one of the quadrupoles before the RFD between -2 mm and 2 mm leads to a relative error less than 5%. The misalignment of the first C-band accelerating section between -2 mm and 2 mm could lead to a relative error up to 10%.

Accuracy losses due to other bunch parameters will be investigated. In particular, the correlations between particle energy and vertical position, and between particle energy and vertical divergences, and the energy spread can affect the bunch length measurements using an RFD. Moreover, the variation of the vertical spot size at screen due to some bunch parameters could be used in order to obtain information about them, by means of a proper measurement technique.

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ORCID iDs

L Sabato https://orcid.org/0000-0003-3998-6762

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